Relativistic space-time positioning: principles and strategies.

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Outline of the talk/1

- Reference frames
- Null frames and null coordinates
- Pulsed em signals and space-time grids
- Reconstruction of null coordinates from proper time measurements
- Accuracy and curvature effects
- Cumulated errors and the continuity problem

Outline of the talk/2

- Sources of pulses
  - Pulsars
    - Radio-pulsars
    - X-ray pulsars
  - Artificial emitters
    - Radio-pulses
    - Laser pulses

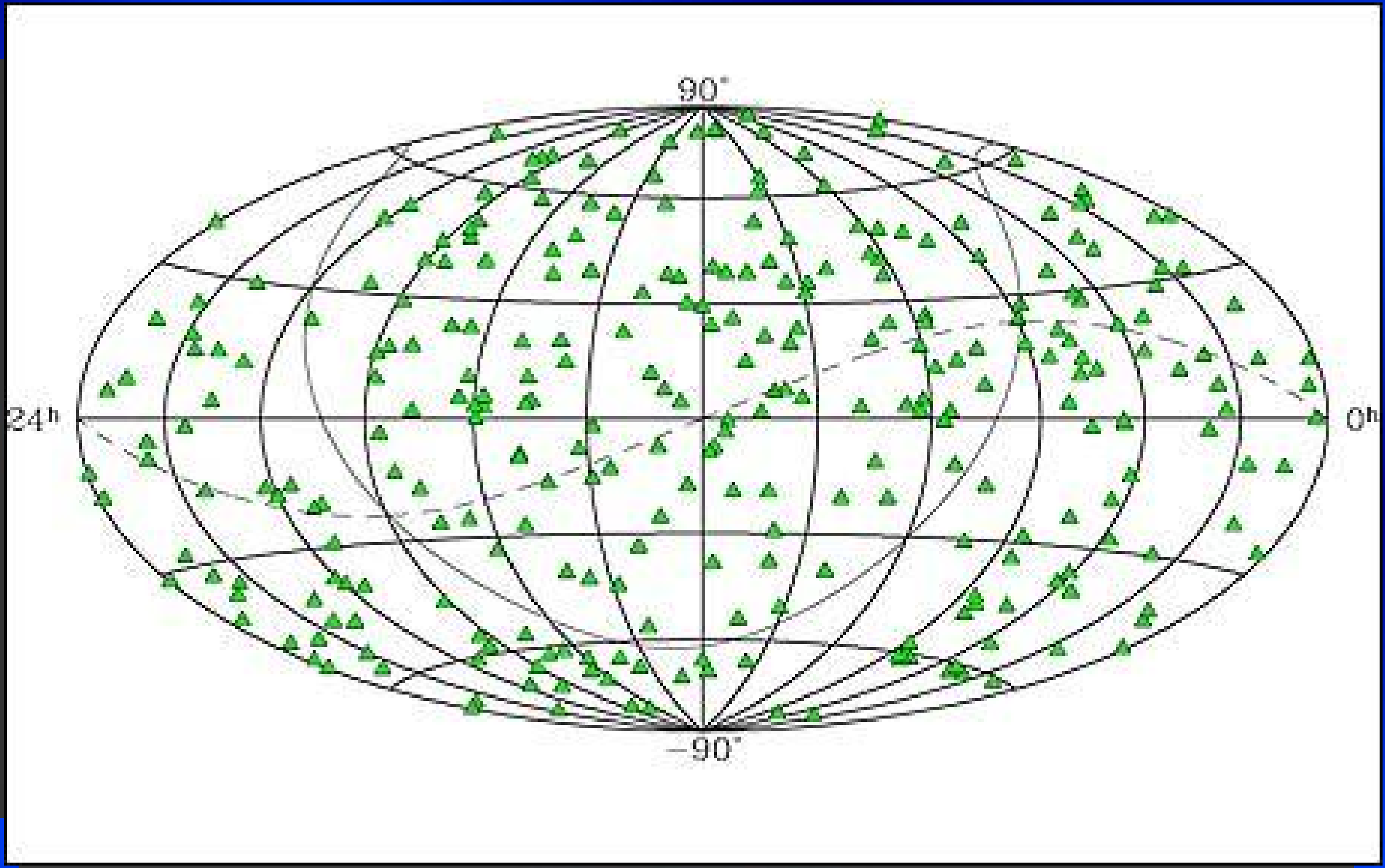
Outline of the talk/3

- Other positioning systems
  - GPS and GPS-like
  - Laser ranging
  - …
- Blended solutions:
  - natural and artificial
  - different approaches together
- Scientific importance of relativistic positioning

Reference frames

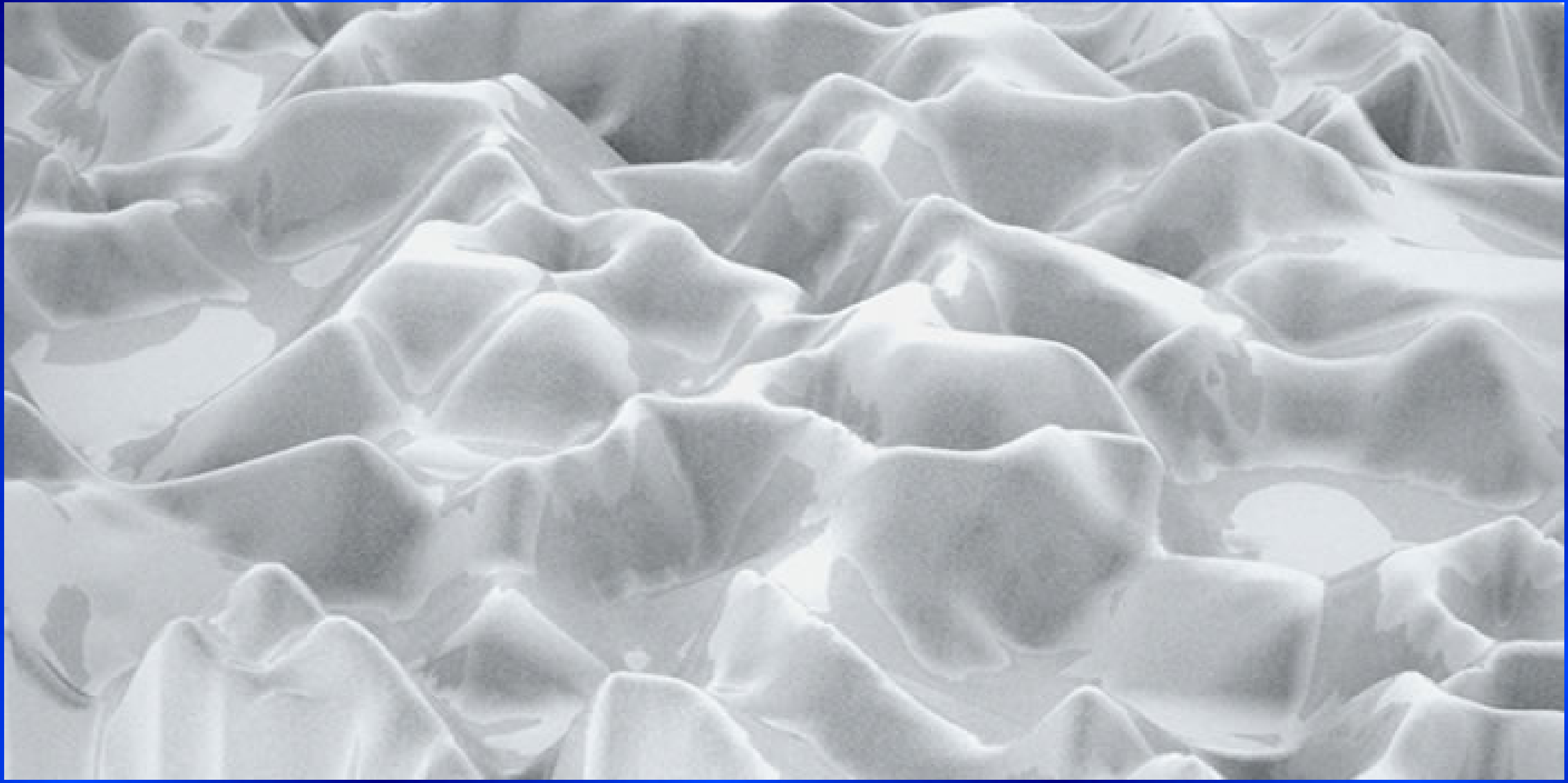
- ICRF: origin in the barycenter of the solar system, directions defined on “fixed” stars: quasars, LB Lacertae objects, galactic nuclei…
  VLBI defined, accuracies ~ 10^{-9} rad.
- ITRF: centered on the earth and corotating with it

Quasars

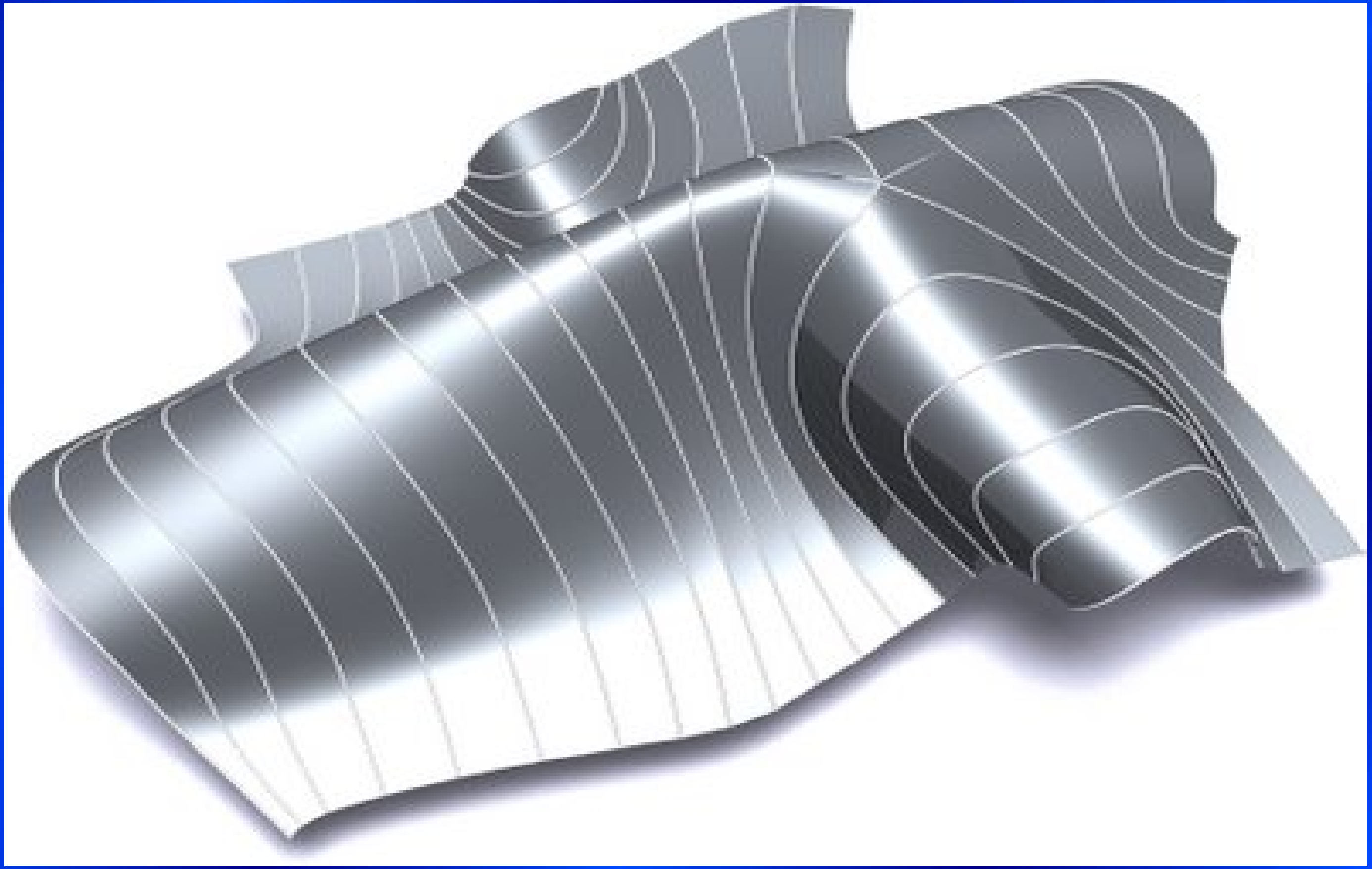


Sloan Digital Survey

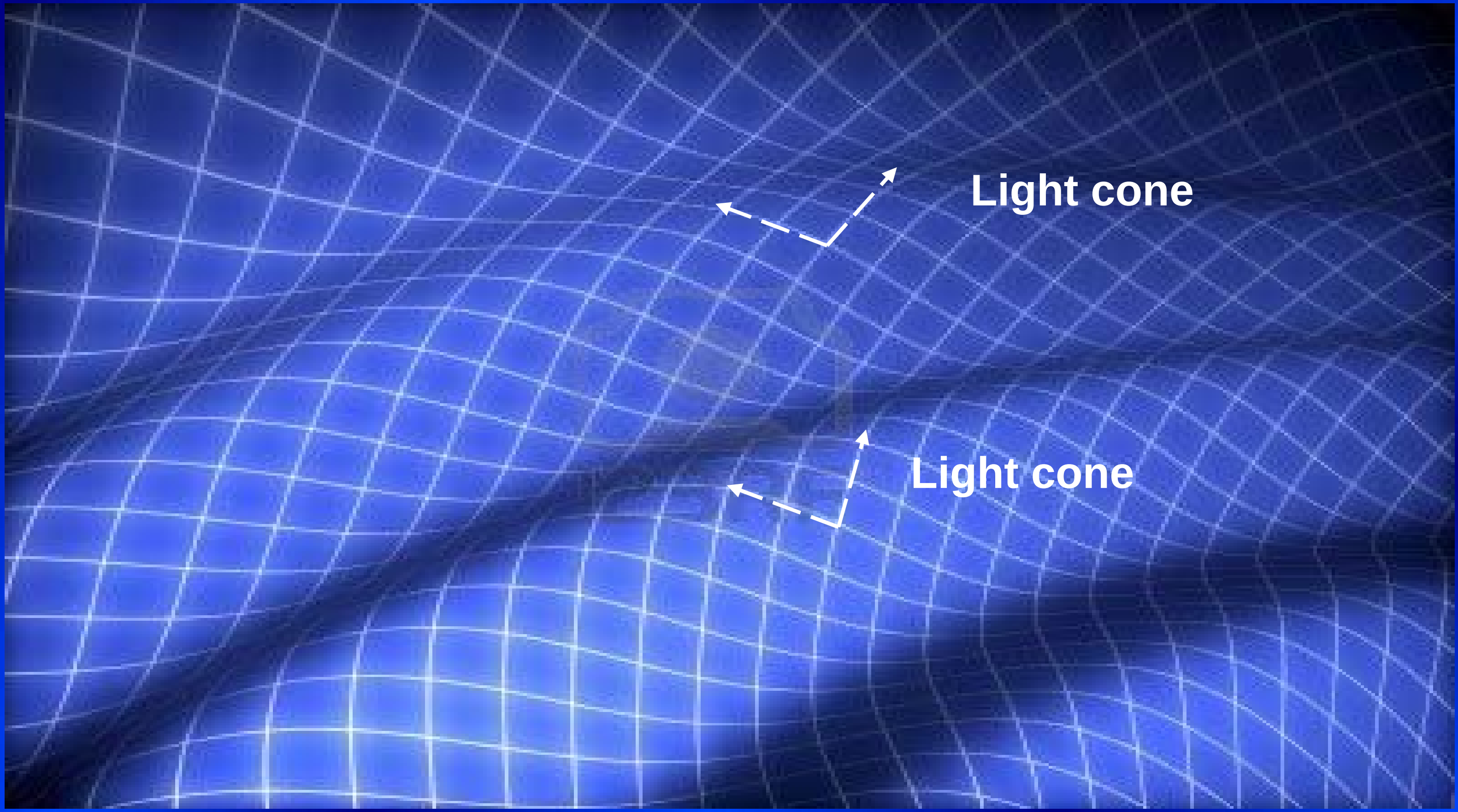
Space-time as a warped four-dimensional manifold with Lorentzian signature



Geodesic curves families accurately map regularly curved space-time patches



Null geodesics



Wave vectors

A null geodesic has a null tangent (wave) vector:

$$\chi = cT\left(1, \cos\alpha, \cos\beta, \cos\gamma\right) = cT\left(1, \hat{n}\right)$$

Period of the signal

Direction cosines

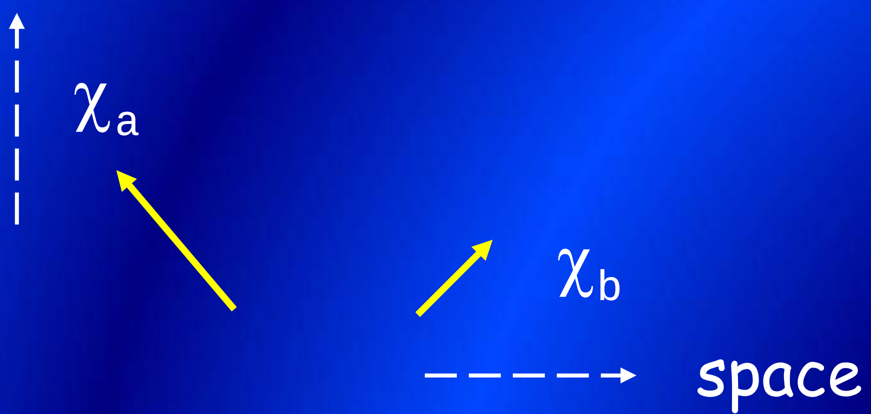
$$\chi^2 = 0$$

Null bases

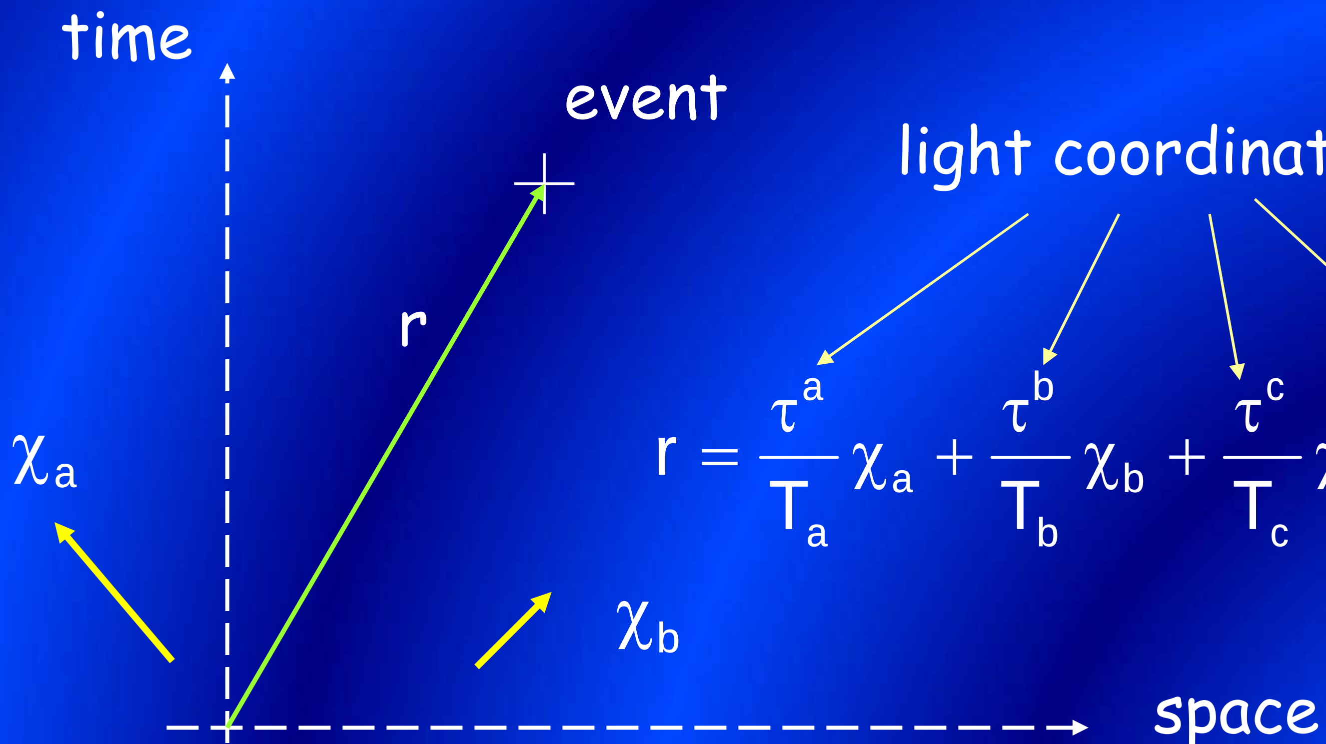
χ_a, χ_b, χ_c, χ_d

Only three vectors needed for events on the light cone; four, out of the light cone

time



Positioning in space-time

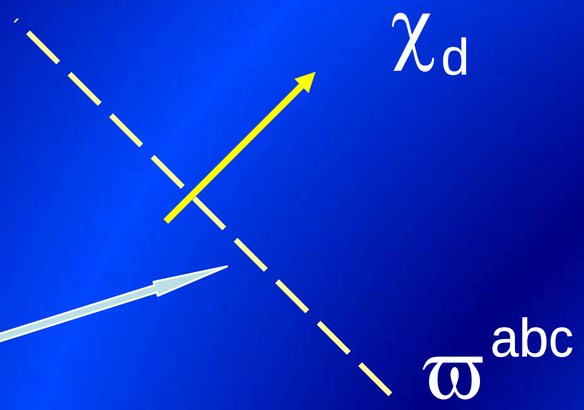


light coordinates

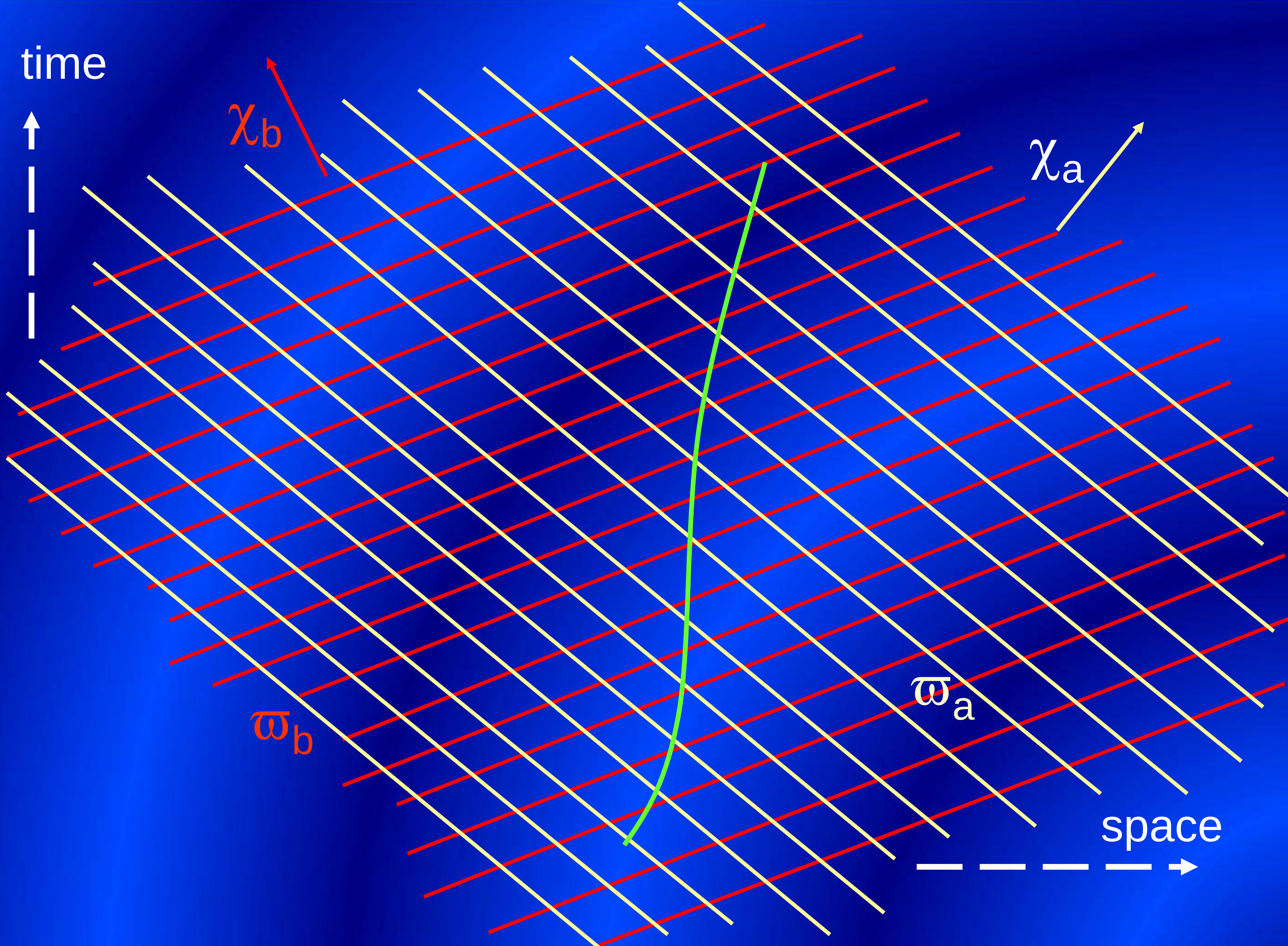
$$r = \frac{\tau^a}{T_a}\chi_a + \frac{\tau^b}{T_b}\chi_b + \frac{\tau^c}{T_c}\chi_c + \frac{\tau^d}{T_d}\chi_d$$

Null wave fronts

hyperplane → $$\varpi^{abc} = \varepsilon^{abcd}\chi_d$$

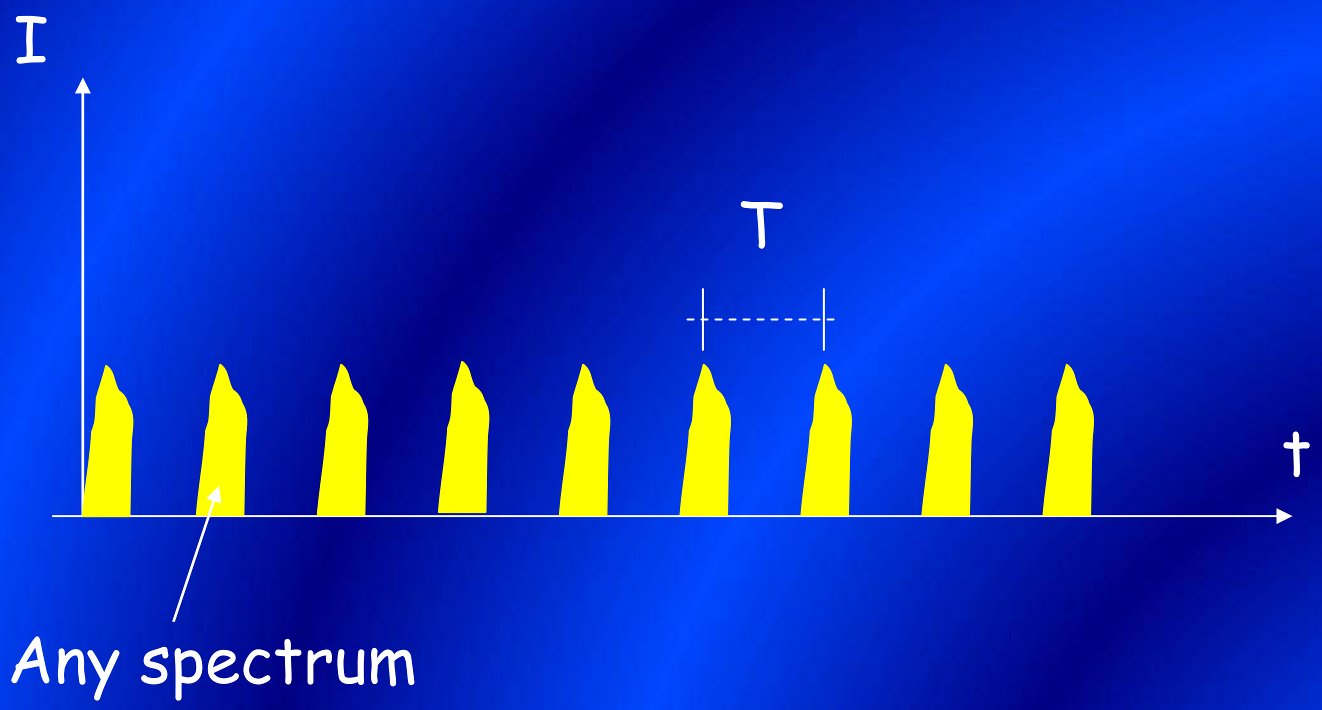


Null three-dimensional hypersurface

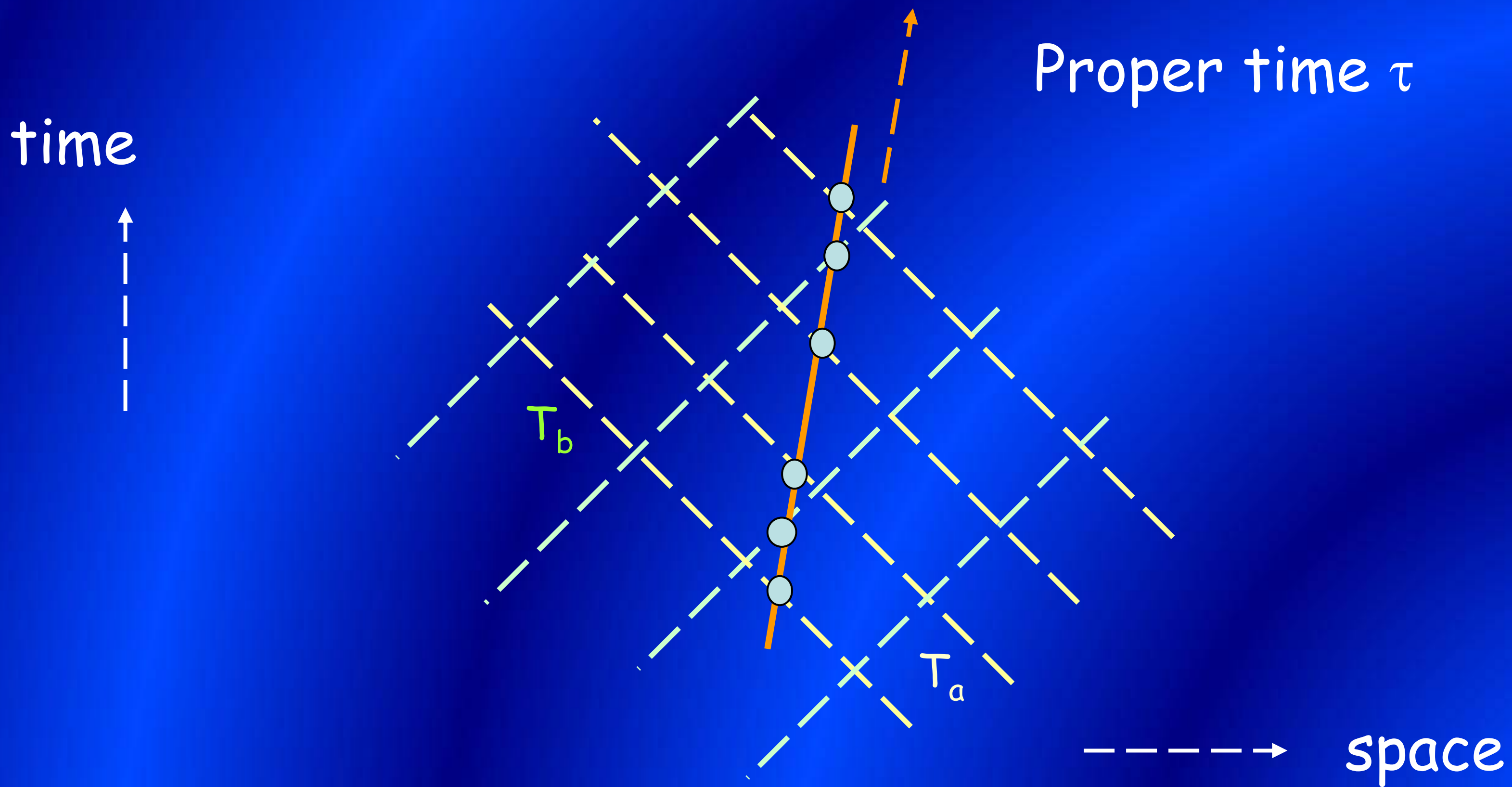


time
χ_b
χ_a
ϖ_a
ϖ_b
space

Discrete sources



Discrete grid, locally uniform motion



Light coordinates of an event

$$\tau_{a,b,c,d} = \left[(n + x) T \right]_{a,b,c,d}$$

integer

From simple linear equations

$$r_i = X_{ai} \chi^a \qquad X_{ai} = \frac{\tau_{ai}}{T_a} = n_{ai} + x_{ai}$$

A linear algorithm

$$r_{ij} = r_j - r_i = \left(X_{aj} - X_{ai}\right)\chi^a = \Delta X_{aij}\chi^a$$

$$\frac{\tau_{ij}}{\tau_{jk}} = \frac{\Delta X_{1ij}}{\Delta X_{1jk}} = \frac{\Delta X_{2ij}}{\Delta X_{2jk}} = \frac{\Delta X_{3ij}}{\Delta X_{3jk}} = \frac{\Delta X_{4ij}}{\Delta X_{4jk}}$$

At least 8 events (two quadruples)

The coordinates

$$x_{a1} = 0,\ x_{b1} = 1 - \frac{\tau_{12}}{\tau_{26}},\ x_{c1} = 1 - \frac{\tau_{13}}{\tau_{37}},\ x_{d1} = 1 - \frac{\tau_{14}}{\tau_{48}}$$

$$x_{a2} = \frac{\tau_{12}}{\tau_{15}},\ x_{b2} = 1,\ x_{c2} = 1 - \frac{\tau_{13}}{\tau_{37}} + \frac{\tau_{12}}{\tau_{37}},\ x_{d2} = 1 - \frac{\tau_{14}}{\tau_{48}} + \frac{\tau_{12}}{\tau_{48}}$$

....

$$\tau_{ij} = \tau_j - \tau_i$$

Difference in arrival times between the j_{th} and the i_{th} event

Uncertainty depends on clock

$$\left|\frac{\delta x}{x}\right| \leq 4\left(\frac{1}{\tau_{i,i+4n}} + \frac{\tau_{i,i+1}}{\tau^{2}{}_{i,i+4n}}\right)\delta\tau$$

As big as allowed by the linearity of the worldline

Accelerated motion

$$x^a = \frac{u^a}{T^a}\tau + \frac{1}{2}\frac{a^a}{T^a}\tau^2 + \ldots$$

Four-velocity

Four-acceleration

Maximum integration time

$$\tau_{\max} = \sqrt{2\left|\frac{u^a}{a^a}\right|\delta\tau}$$

A gravitational field

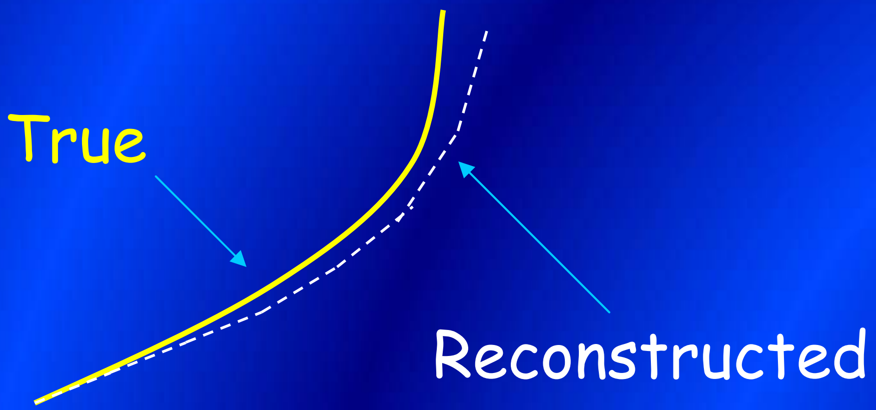
The gravitational field shows up when:

$$\left| u \cdot \nabla \Phi \right| \geq 4 \frac{\delta\tau}{\tau^2}$$

Gravitational potential

Projection on the tangent space-time

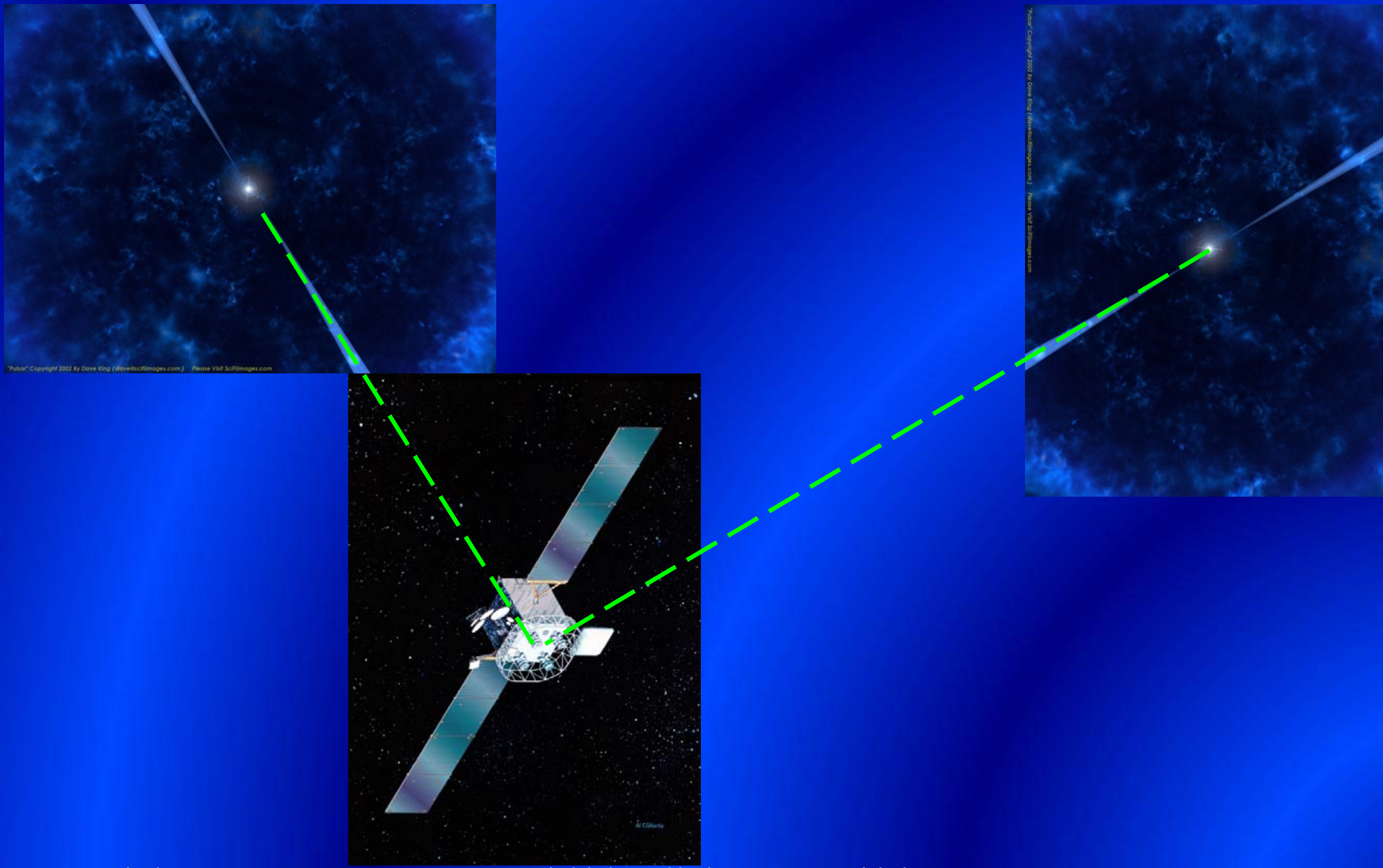
A problem



The error tends to grow with time

Need for periodic independent position fixing

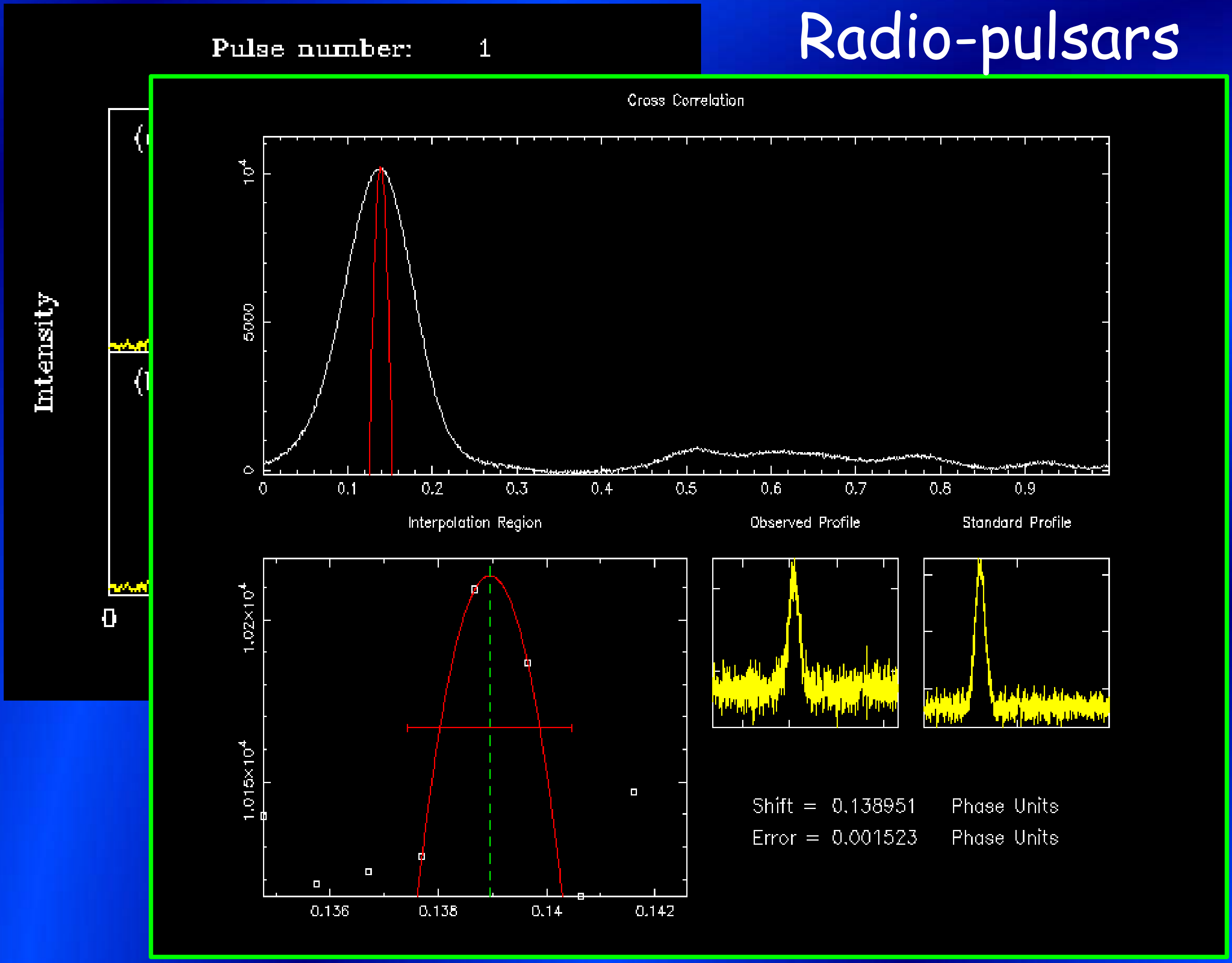
Pulsars as beacons



Pulsars

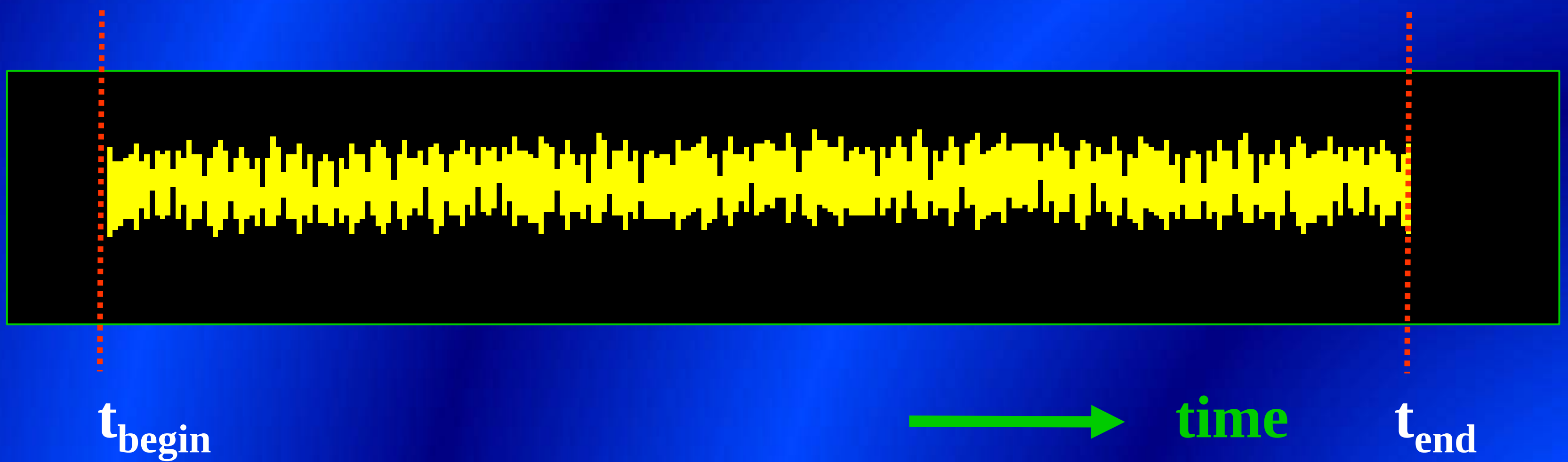
- ~ 2000
- A few hundreds X-ray pulsars
- Millisecond pulsars: 144
  - Isolated 57
- Extremely good clocks
- Almost fixed positions in the sky but uneven distribution.

Radio-pulsars

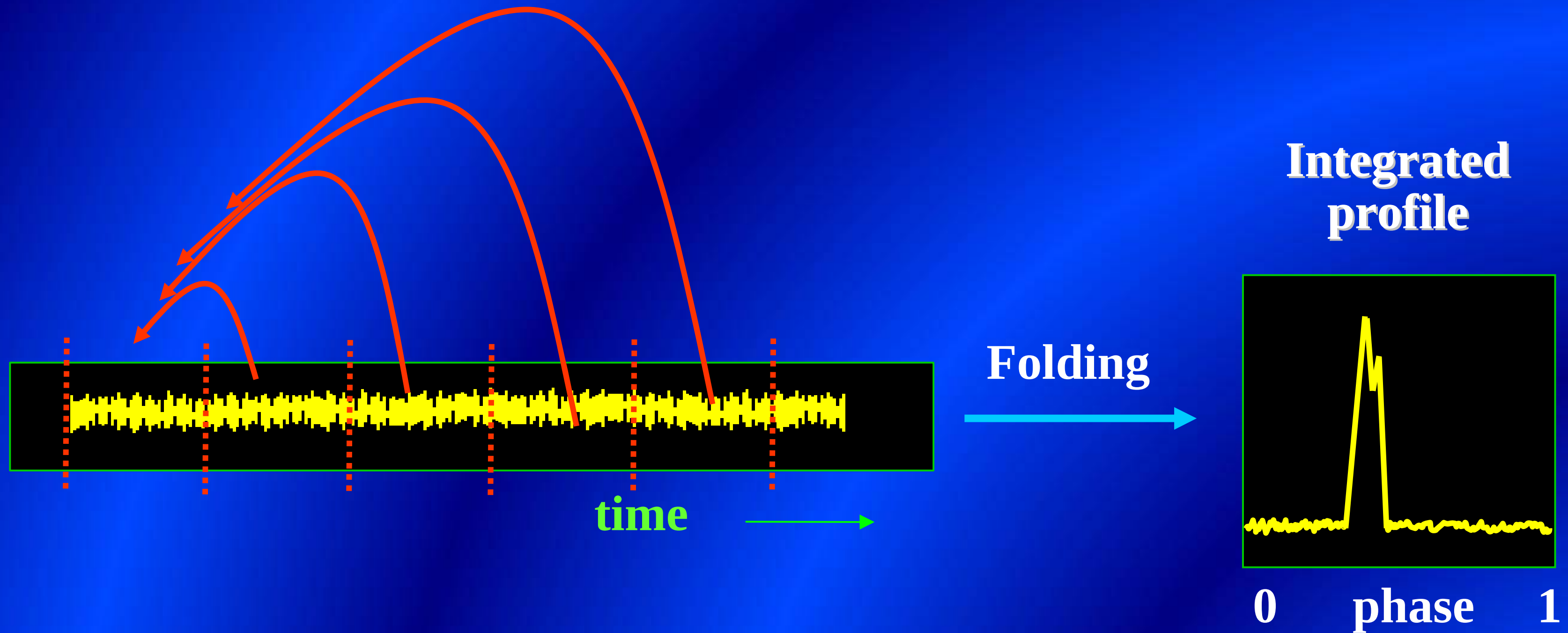


Inconveniencies (radio domain)

- Extremely week signals: N/S ~ 50 dB
- Need for long enough integration time



Integration and folding



We already know the pulsars we use and the corresponding templates are available

Proper motion and period change

- Typical change of the position in the sky: $\frac{d\alpha}{dt} \approx 10^{-6} \left(\frac{100 \text{ pc}}{\text{distance}} \right) \text{rad/year}$
- Decay of the period: $\frac{\delta T}{T} \approx 10^{-15} \div 10^{-21}$
- Stability over months
- The change rates are known
- Redundancy for random phenomena (glitches)

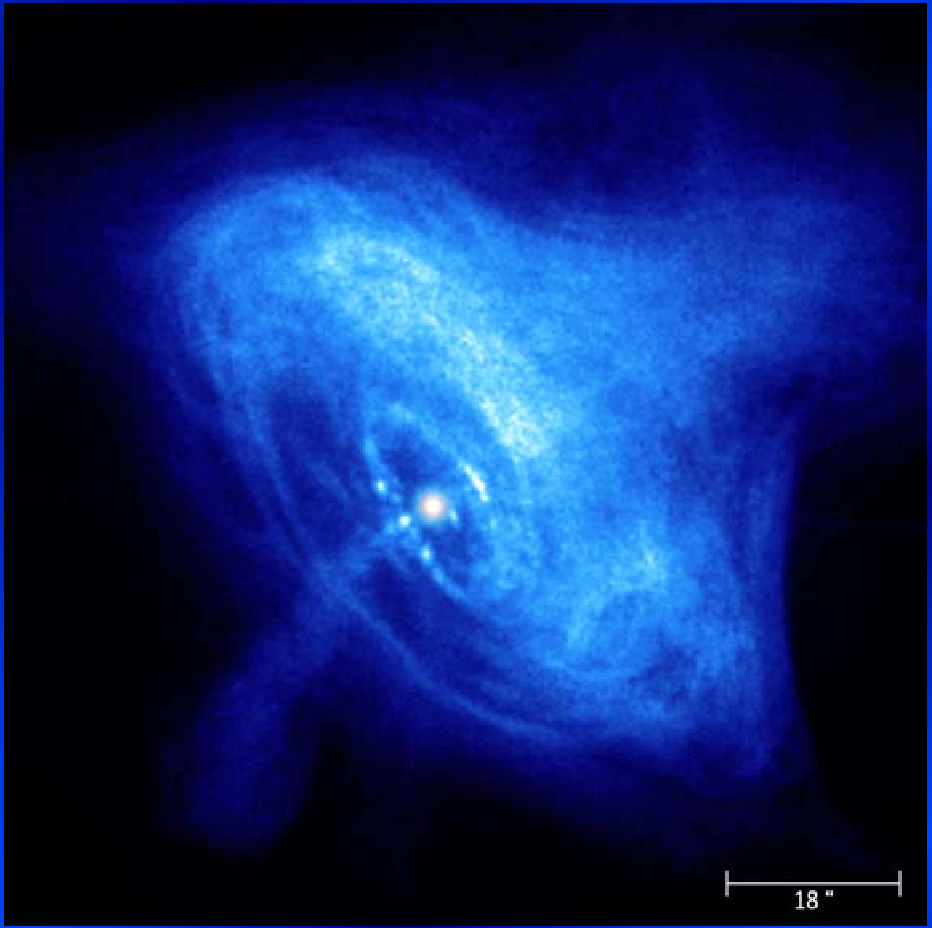
Practical problems: huge antennas

$\text{Area} \geq \sim 10\ m^2$

Not easy to look at 4 or more sources at a time: line of sight controlled by interferencial techniques

X-ray emitters

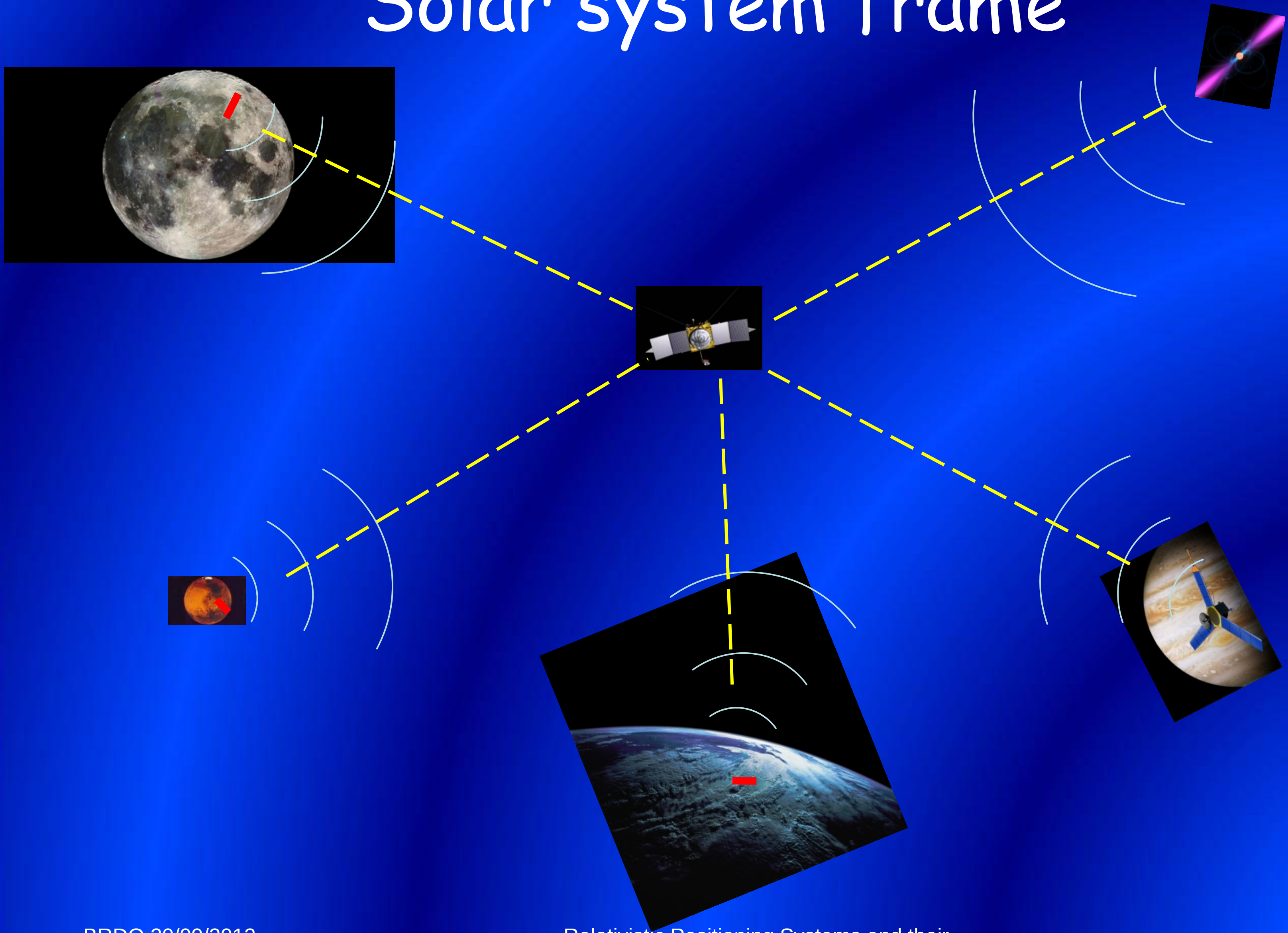
• useful out of the atmosphere only
• small antennas required
• more than one antenna required
• smaller background noise



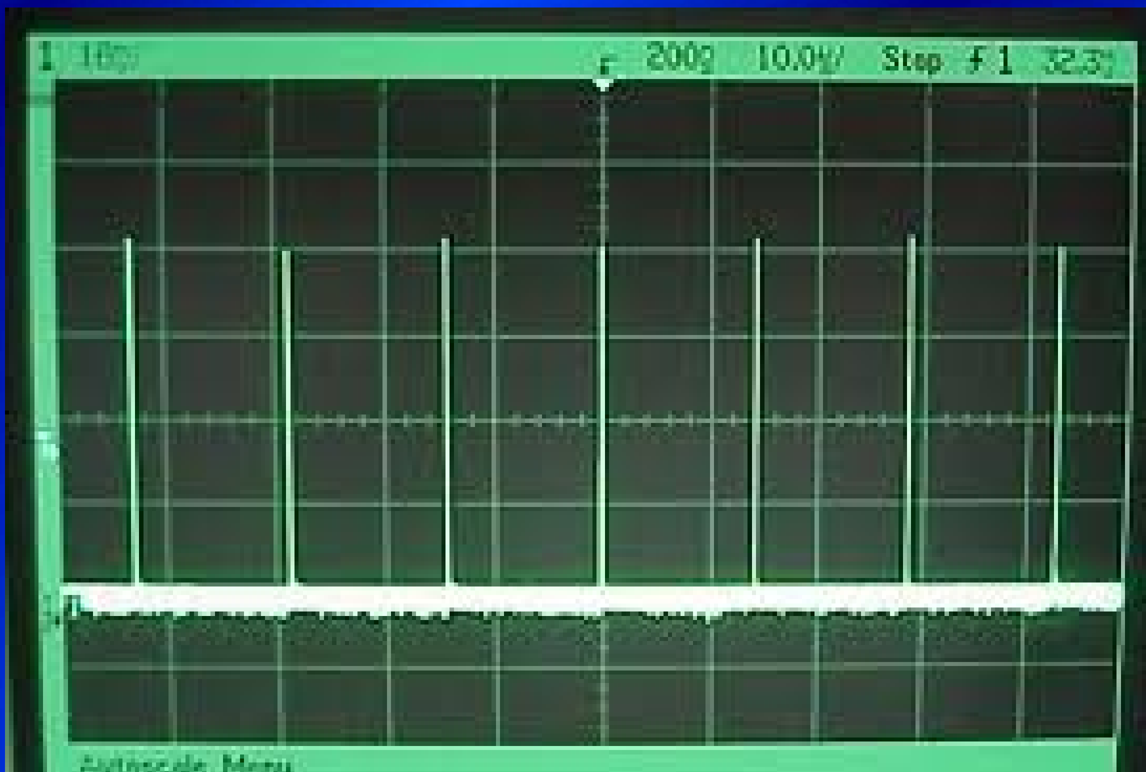
Artificial sources

- Atomic clocks on spacecrafts or celestial bodies
- Wide frequency range: $10^3 \div 10^{11}$ Hz
- Well defined shape and intensity of the signal
- Need to know the worldline of the emitter
- Small enough acceleration

Solar system frame

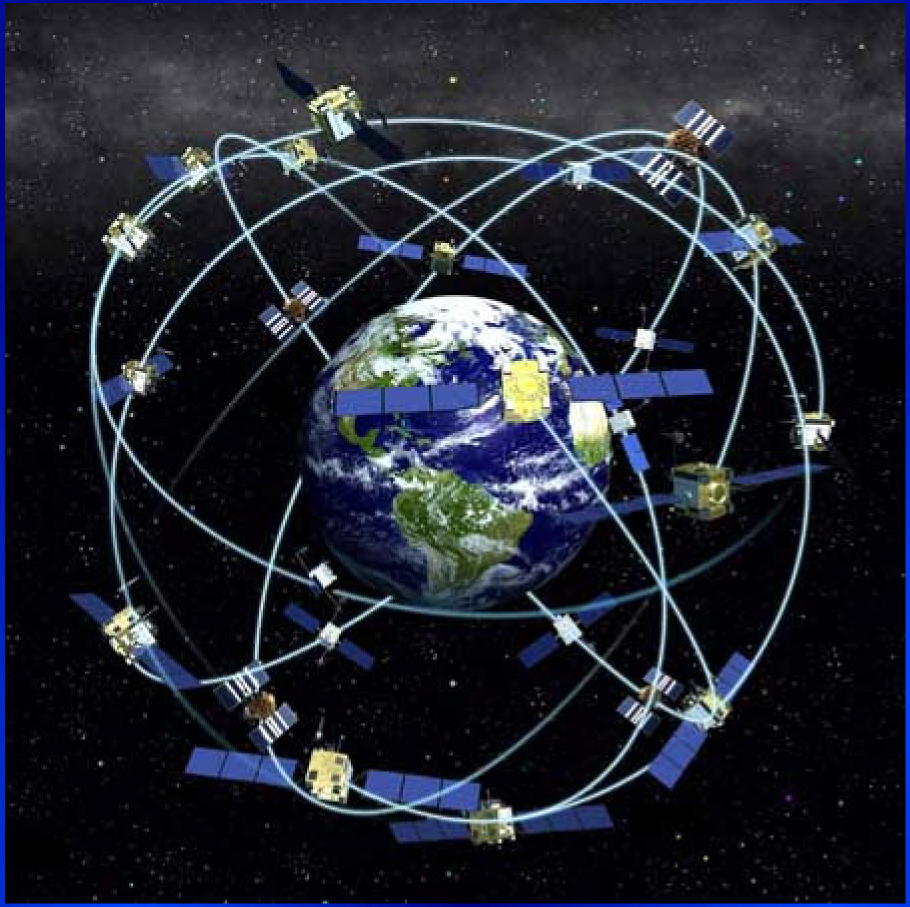


LASER pulses



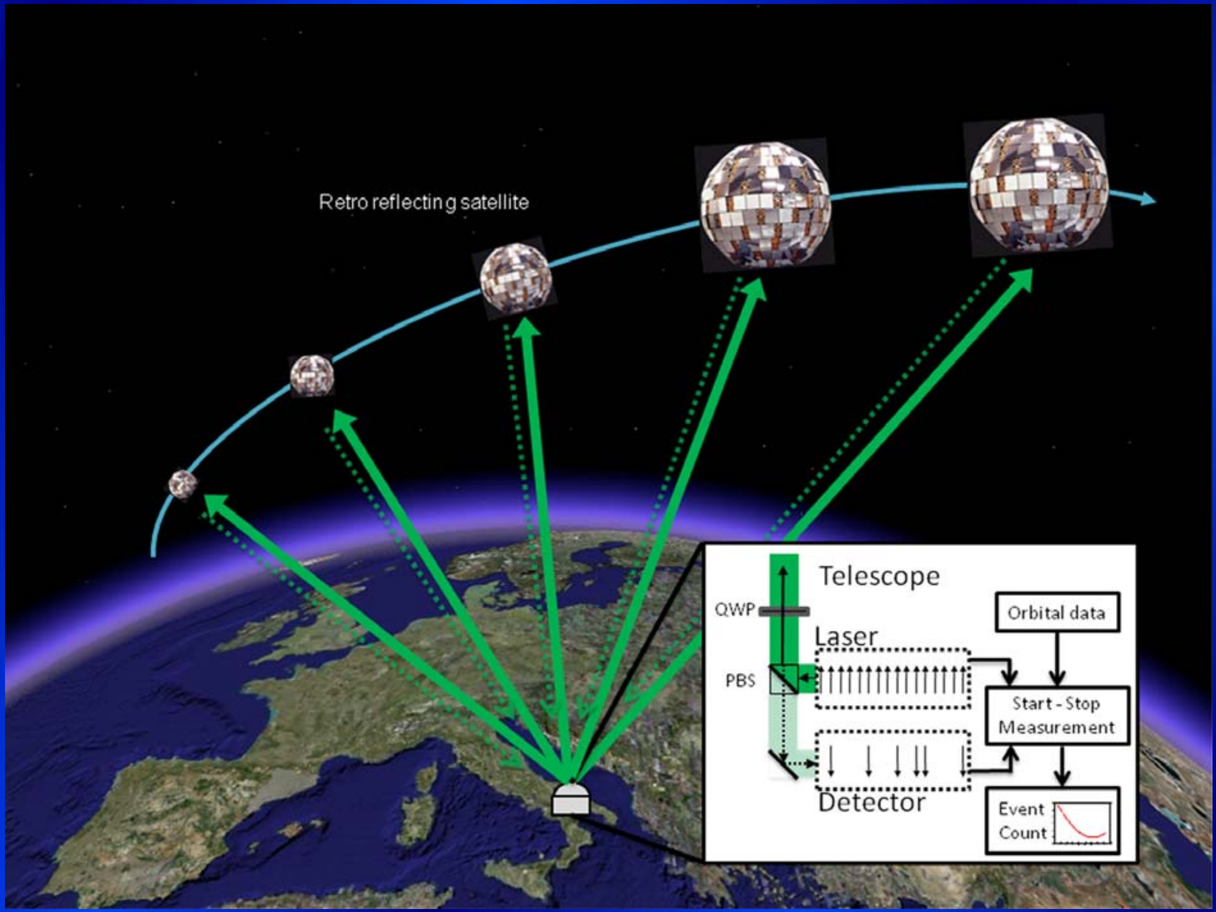
- ~ kHz pulses
- length of a pulse: tens of ps
- highly collimated beams ~ 1 mrad

Other positioning: GPS



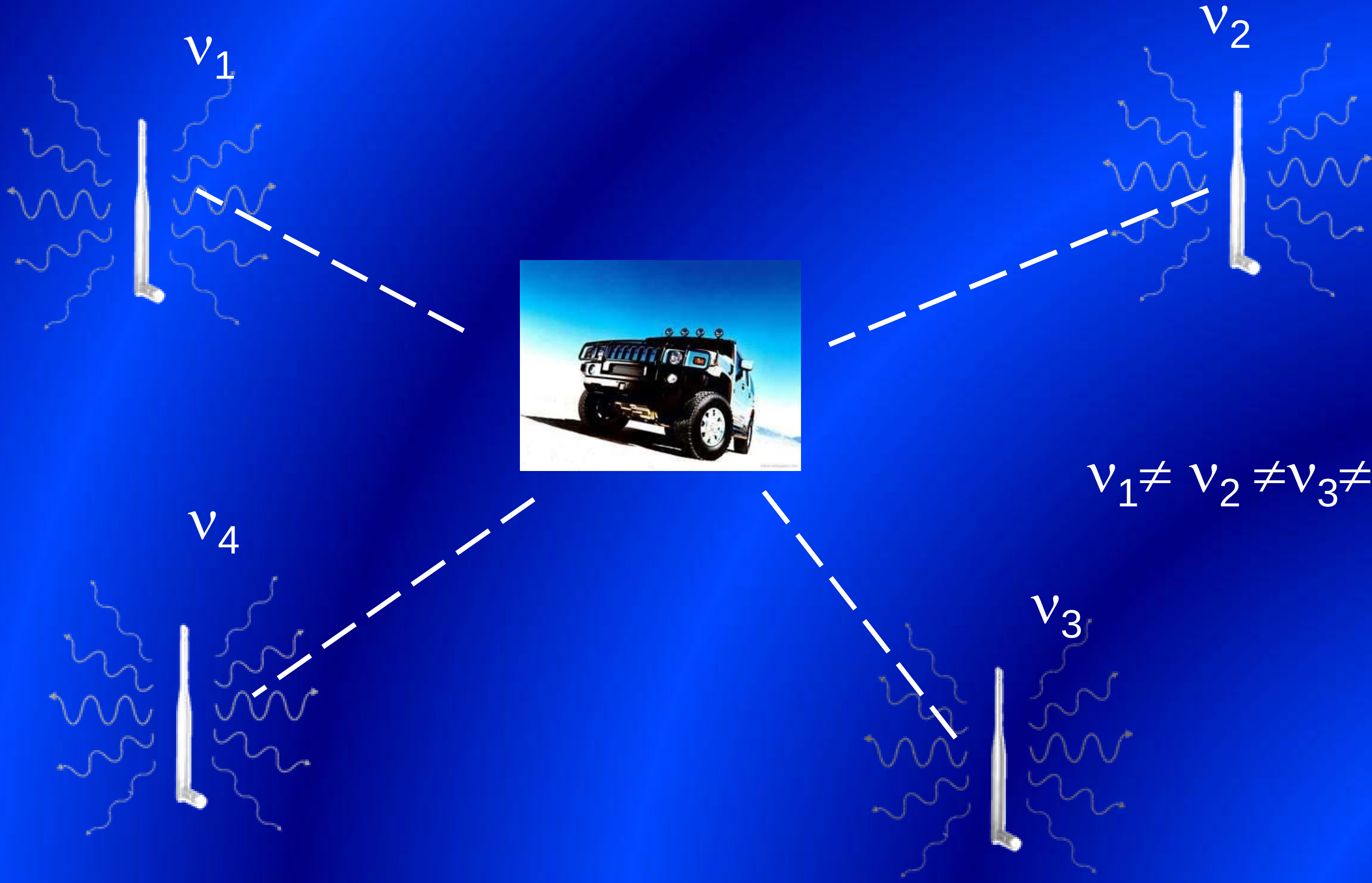
- Based on time of flight measurement
- requires synchronized clocks
- range determined by trial and error
- relativity introduced as corrections

LASER ranging



Distances with millimeter accuracy

Mapping a limited region on Earth

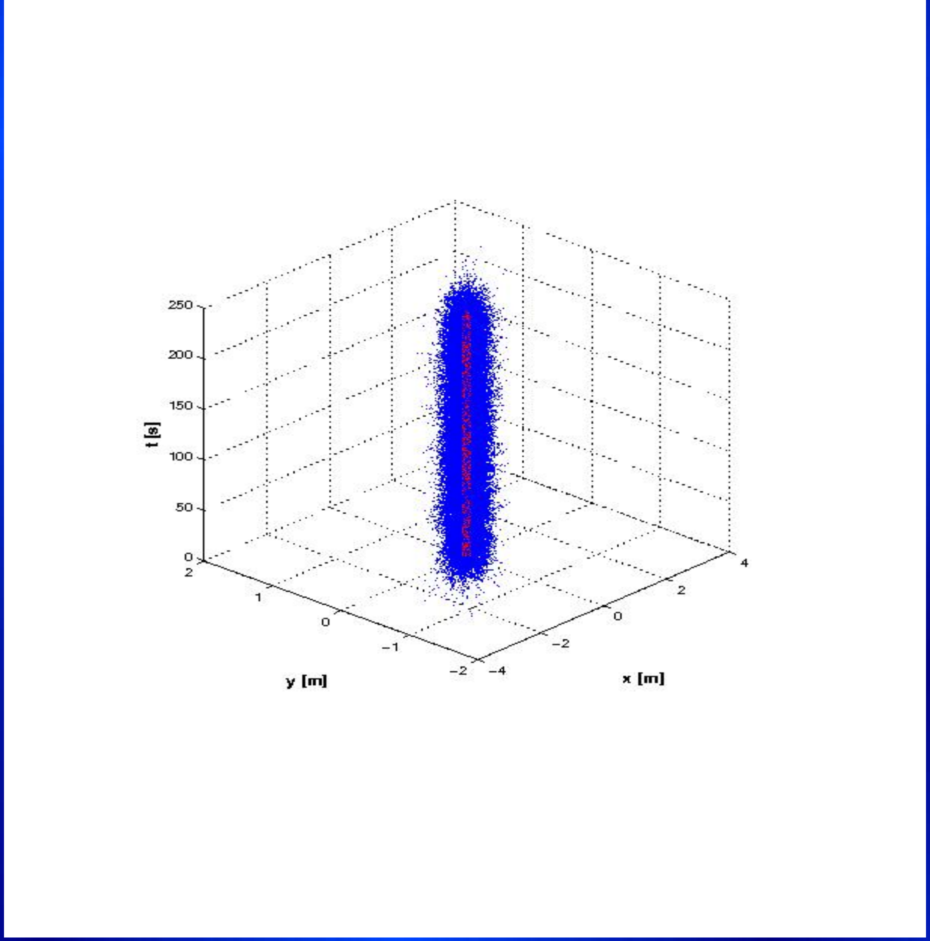


$\nu_1 \neq \nu_2 \neq \nu_3 \neq \nu_4$

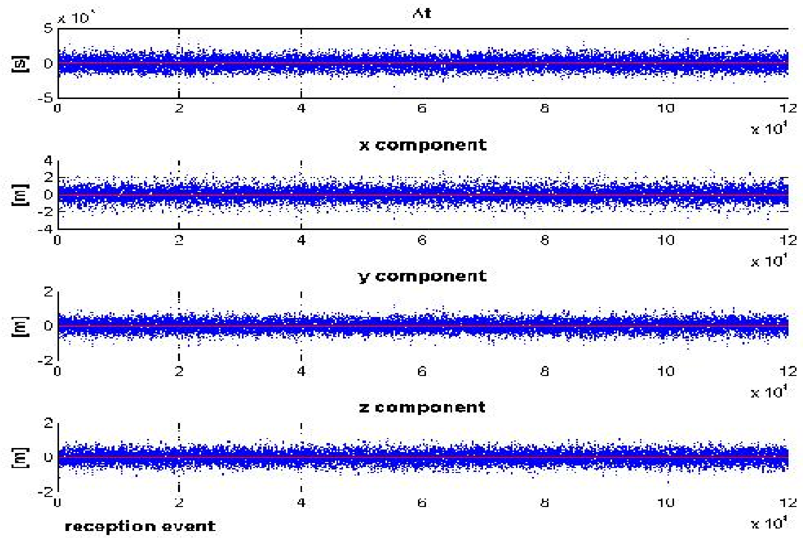
An exercise: four real pulsars

Pulsar	T (ms)	Elong (°)	Elat (°)
J1730-2304	8.123	263.19	0.19
J2322+2057	4.808	0.14	22.88
B0021-72N	3.054	311.27	−62.35
B1937+21	1.558	301.97	42.30

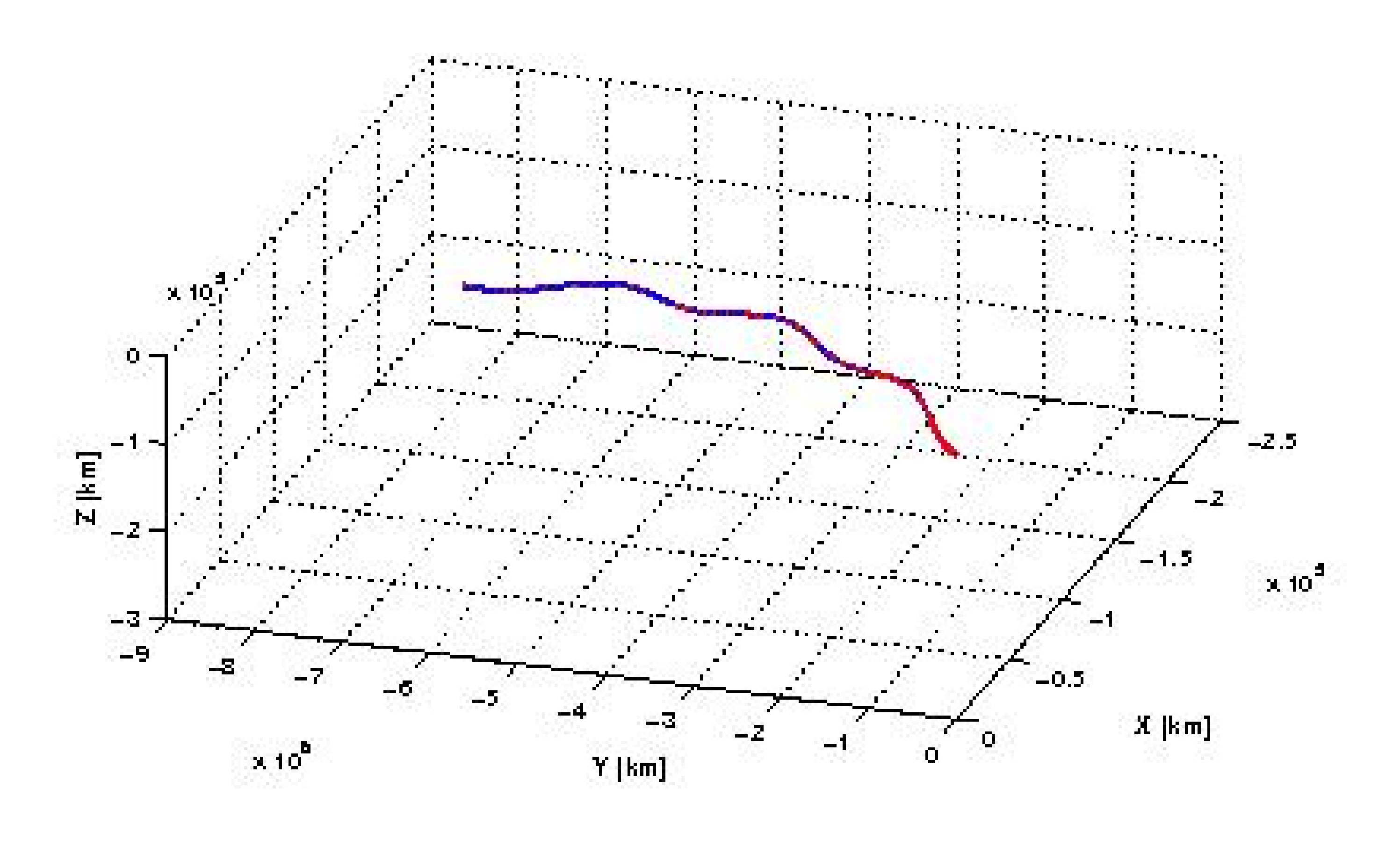
Static observer



Uncertainties

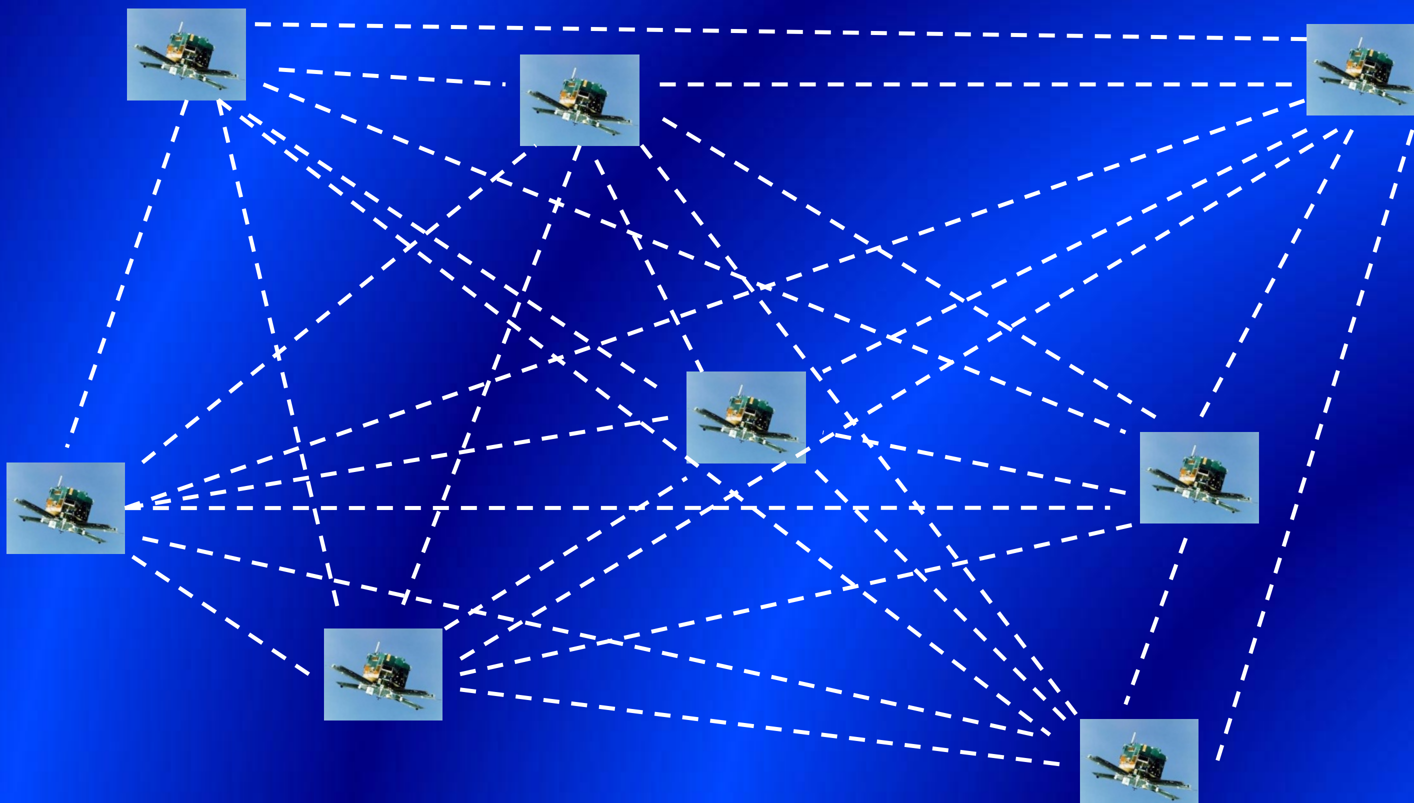


Eppur si muove



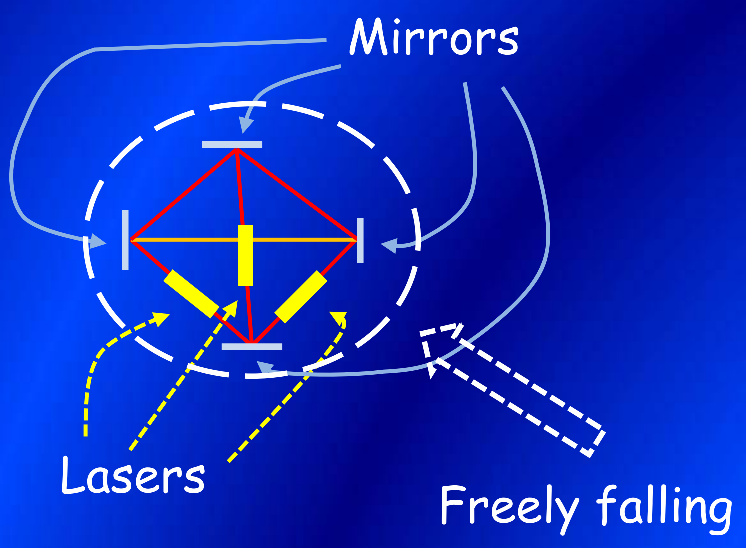
Positioning, General Relativity, time measurement and science in space

Self positioning of a swarm of satellites



Space-time geodesy

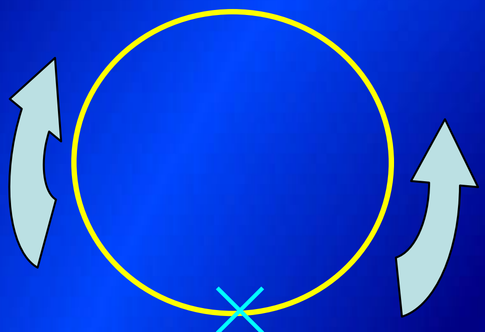
Locally probing the curvature



Tetrahedron with four triangular ringlasers

The effect on frequency

Closed loop (shape irrelevant)



Counterrotating light beams

Time of flight difference

$$\delta T = T_+ - T_- = -2\oint \frac{g_{0\phi}}{g_{00}} d\phi \neq 0$$

$$\delta\tau_0 = -2\sqrt{g_{00}}\oint \frac{g_{0\phi}}{g_{00}} d\phi \neq 0$$

$$\Delta f = 4\frac{S}{\lambda P}\left[\frac{\omega}{c}\left(1+\frac{3}{2}\frac{GM}{c^2R}-\frac{3}{4}\sqrt{\frac{GM}{c^2R}}+\frac{5}{2}\frac{\omega^2R^2}{c^2}\right)\hat{n}_a\cdot\hat{n}_S-\frac{1}{2}\frac{(GM)^{3/2}}{c^3R^{5/2}}\hat{n}_\theta\cdot\hat{n}_S-\left(\frac{GJ}{c^3R^3}\right)\hat{n}_\vartheta\cdot\hat{n}_S\right]$$

Beat frequency (equatorial orbit)

Corotating device

$$\omega = \Omega = \sqrt{G\frac{M}{R^3}}$$

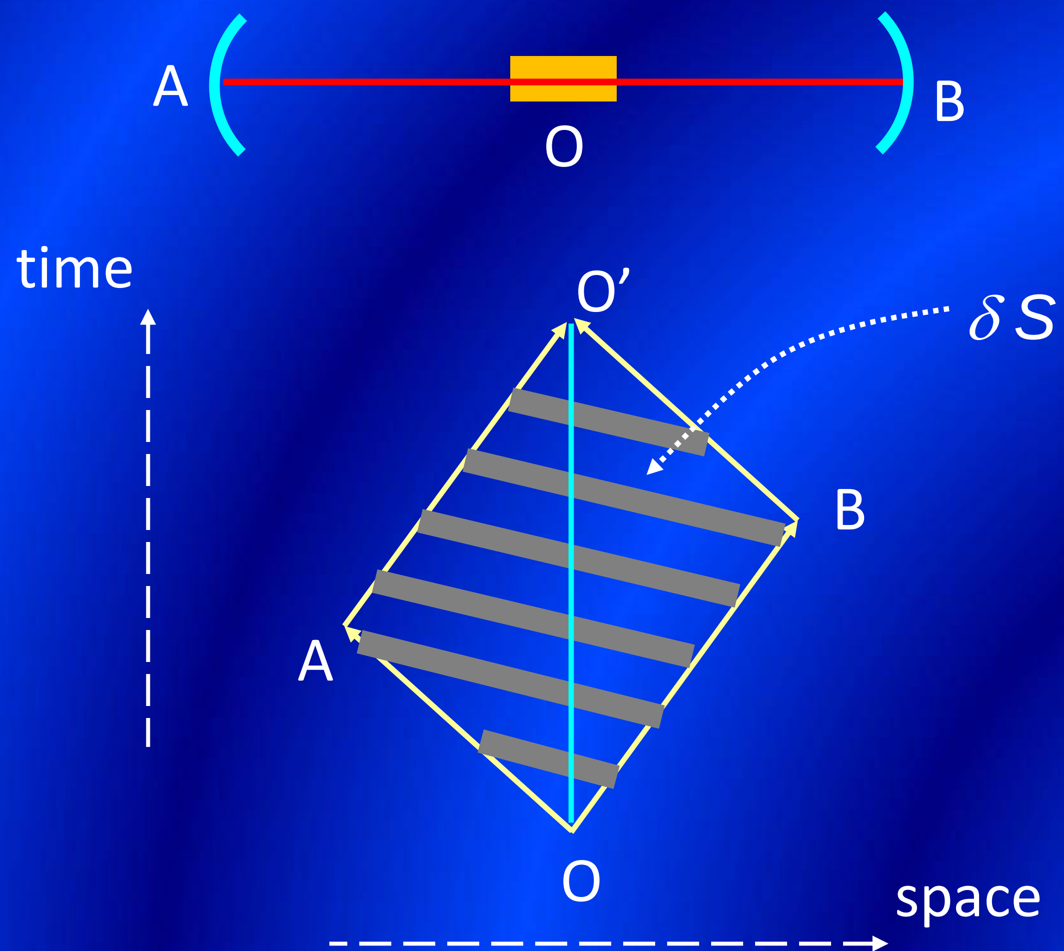
$$\Delta f = 4\frac{S}{\lambda PR}\sqrt{\frac{GM}{R}}\left[\left(1-\frac{3}{4}\sqrt{\frac{GM}{c^2R}}+4\frac{GM}{c^2R}\right)\hat{n}_a\cdot\hat{n}_S-\frac{1}{2}\frac{GM}{c^2R}\hat{n}_\theta\cdot\hat{n}_S-\chi\left(\frac{\sqrt{GMR}\,\Omega_\oplus}{c^2}\right)\hat{n}_\vartheta\cdot\hat{n}_S\right]$$

$\sim 10^{-4}$

$\sim 10^{-9}$

$\sim 10^{-12}$

A linear cavity



The effect of curvature and anisotropy

$$\delta F^{\mu\nu} = \left(R^{\mu}{}_{\varepsilon 0 i} F^{\varepsilon\nu} + R^{\nu}{}_{\varepsilon 0 i} F^{\mu\varepsilon}\right)\delta S^{0i}$$

Electromagnetic tensor

Riemann tensor

Space-time area
spanned by the cavity

The result depends on the orientation of the cavity

Linearized Riemann tensor. An example:

$R^{\phi}{}_{r0\theta}$

Length of the cavity

$$\delta F^{\vartheta\phi} \cong \left[\left(\frac{(GM)^{3/2}}{c^3 R^{7/2}} - 3\frac{GJ}{c^3 R^4}\right)\frac{\cos\vartheta}{\sin^2\vartheta}\right]\frac{l^2}{R} F^{\vartheta r}$$

Change in the radial component of the magnetic field

East-West component of the magnetic field

Orbital gyro

Arrival times difference

Equatorial orbit

$$\Delta t_0 \cong -6\frac{R}{c}\left[\left(24\sqrt{\frac{2}{\sqrt{3}}}+3\sqrt{\frac{\sqrt{3}}{2}}\right)\left(\frac{GM}{c^2R}\right)^{\frac{3}{2}}-2\frac{\pi}{3}\frac{GM}{c^2R}+4\frac{GJ}{c^3R^2}+\sqrt{2\sqrt{3}}\sqrt{\left(\frac{GM}{c^2R}\right)}\right]$$

Pulses

Beat frequency

Equatorial orbit

$$\Delta f = \frac{c^2}{\lambda P}\Delta t_0 = \frac{c^2}{3\sqrt{3\lambda R}}\Delta t_0$$

Length of the loop

Conclusion

- Light is an intrinsically relativistic probe of space-time
- Proper time measurements between the arrivals of successive electromagnetic pulses are a promising means for an autonomous space navigation system
- Timing plus space-time geodesy can help in reconstructing the average corvature in small regions of space and can play a role in testing GR effects.

• ML. Ruggiero, E. Capolongo, A. Tartaglia, *Relativistic Positioning System by Means of Pulsating Sources for Navigation through the Solar System and beyond*, in "Solar System: Structure, Formation and Exploration", ed. Matteo Rossi, Nova Science Publishers Inc. ISBN: 9781621000570, 2011

• ML. Ruggiero, E. Capolongo, A. Tartaglia, *Pulsars as celestial beacons to detect the motion of the Earth*, IJMPD, **20**, 1025-1038 (2011).

• A. Tartaglia, ML. Ruggiero, E. Capolongo, *A null frame for spacetime positioning by means of pulsating sources*, Advances in Space Research, **47**, 645-653 (2011).

• A. Tartaglia, M.L. Ruggiero, E. Capolongo, *A relativistic navigation system for space*, ACTA FUTURA, Advanced Concept Team, European Space agency, **4**, 33-40, 2011

• **A.** Tartaglia, *Emission Coordinates for the Navigation in Space*, Acta Astronautica, **67**, 539-545, 2010