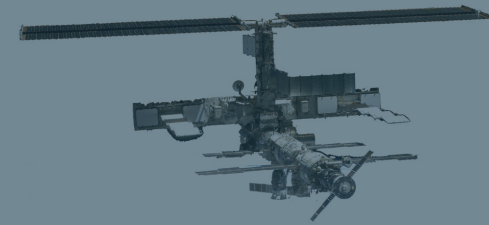


ATOMIC CLOCKS : NEW PROSPECTS IN METROLOGY AND GEODESY



**Workshop - Relativistic Positioning Systems
and their Scientific Applications**

Brdo near Kranj, Slovenia

September 2012, 19-21st

Pacôme DELVA





■ Metrology

- Toward a new definition of the second
- Improving the timescale
- Going in space

■ Geodesy

- Relativistic geodesy
- Using clocks to measure the geopotential
- The relativistic geoid



■ Metrology

- Toward a new definition of the second
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- Going in space

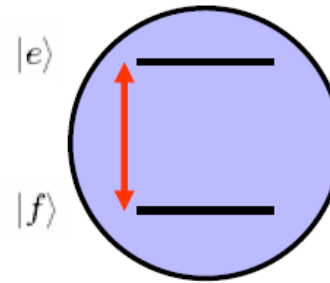
■ Geodesy

- Relativistic geodesy
- Using clocks to measure the geopotential
- The relativistic geoid



→ Deliver a signal with stable and universal frequency

$$\hbar\omega_{ef} = E_e - E_f$$



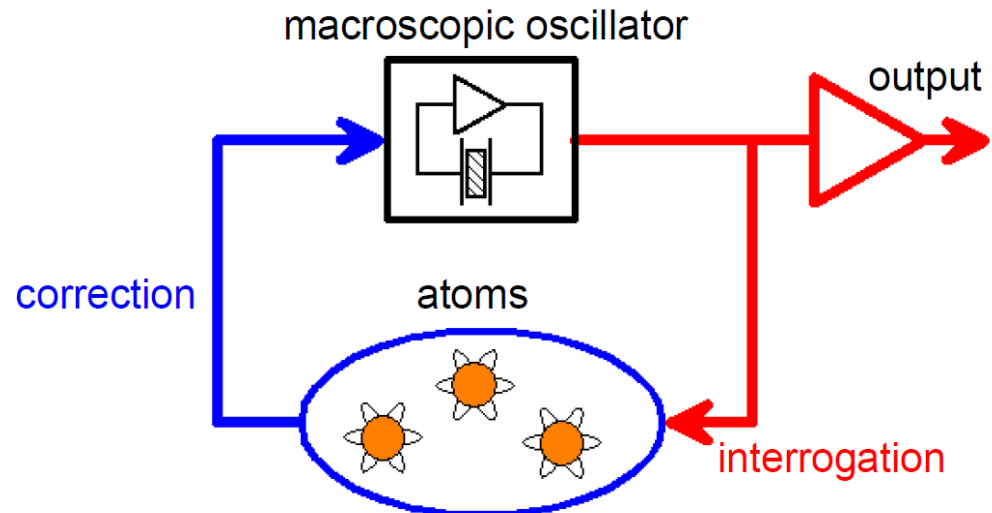
$$\omega(t) = \omega_{ef} \times (1 + \epsilon + y(t))$$

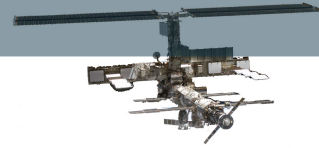
ϵ : fractional frequency offset

Accuracy: overall uncertainty on ϵ

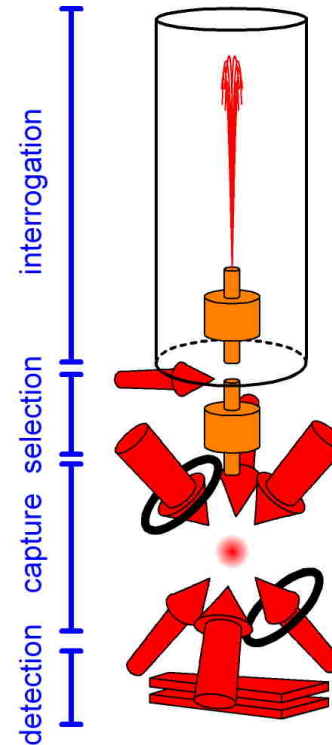
$y(t)$: fractional frequency fluctuations

Stability: statistical properties of $y(t)$, characterized by the Allan variance

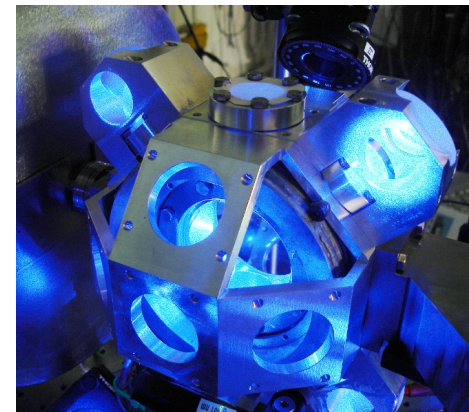
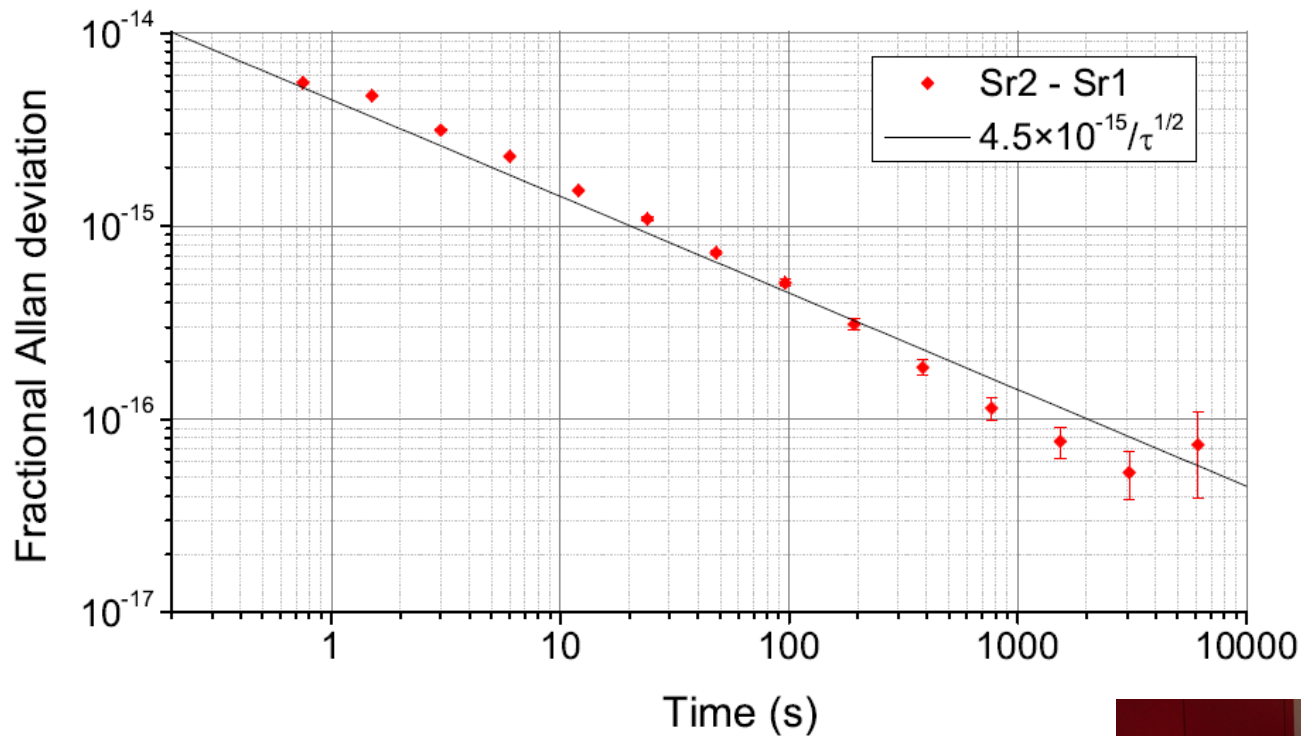
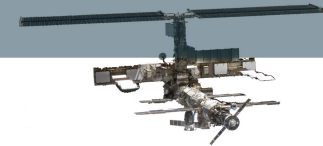




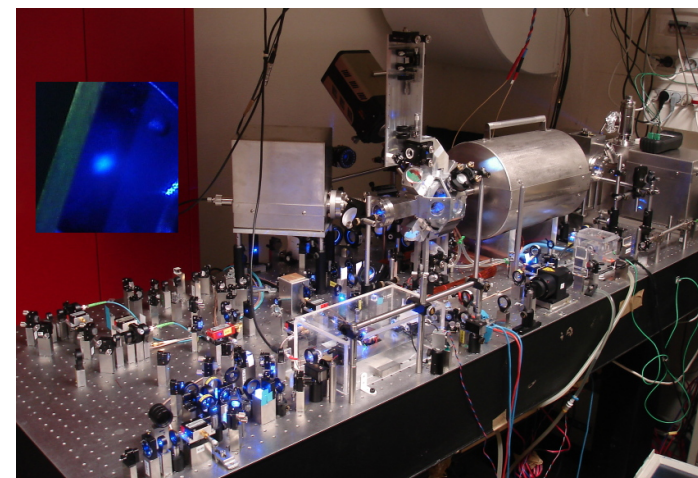
- CGPM 1967 : « the second is the duration of 9192631770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the caesium 133 »
- Cs fountains :
 - Primary Frequency Standards (PSF), only a few worldwide
 - Accuracy (relative frequency error) : $2-3 * 10^{-16}$
 - Stability : few $10^{-14}/\sqrt{\tau}$, i.e. around 10^{-16} @ 1d
 - Local frequency comparisons effectively achieve such noise levels



SYRTE Rb/Cs dual fountain FO2



- $4.5 * 10^{-15} / \sqrt{\tau}$
- Goes below 10^{-16} after 1000 s
(J. Lodewyck et al.)



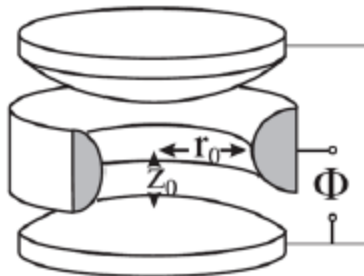


■ **Single ion clock :**

- Very few interaction with the trap → good accuracy
- Al+ NIST : $9 * 10^{-18}$
- Stability ($\sim 3 * 10^{-15} / \sqrt{\tau}$) limited by the shot noise (only one ion)

■ **Developments**

- NIST : Al+, Al+, Hg+
- Europe : PTB, NPL, ...
- France : Provence University (Ca+)

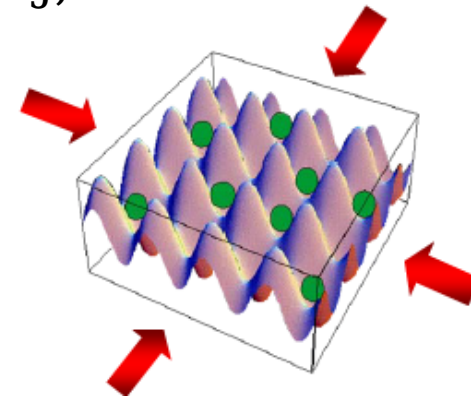


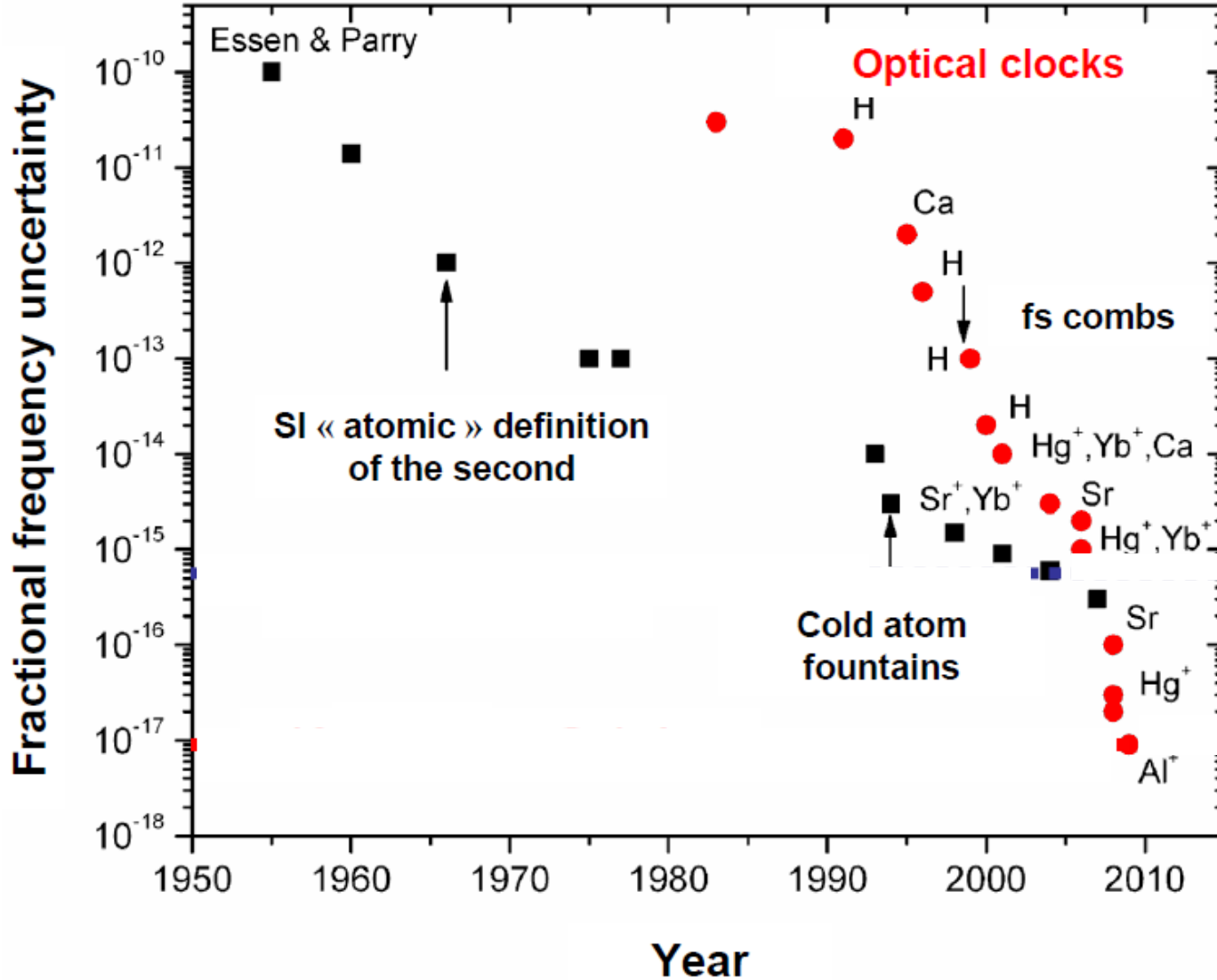
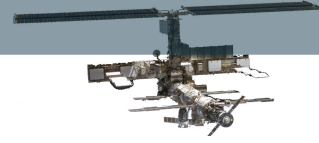
■ **Neutral atoms**

- Intense trap but controlled
- Accuracy $\sim 2 * 10^{-16}$ but in constant improvement
- High number of atoms (10^4) → good ultimate stability (actually $\sim 2 * 10^{-15} / \sqrt{\tau}$)

■ **Developments**

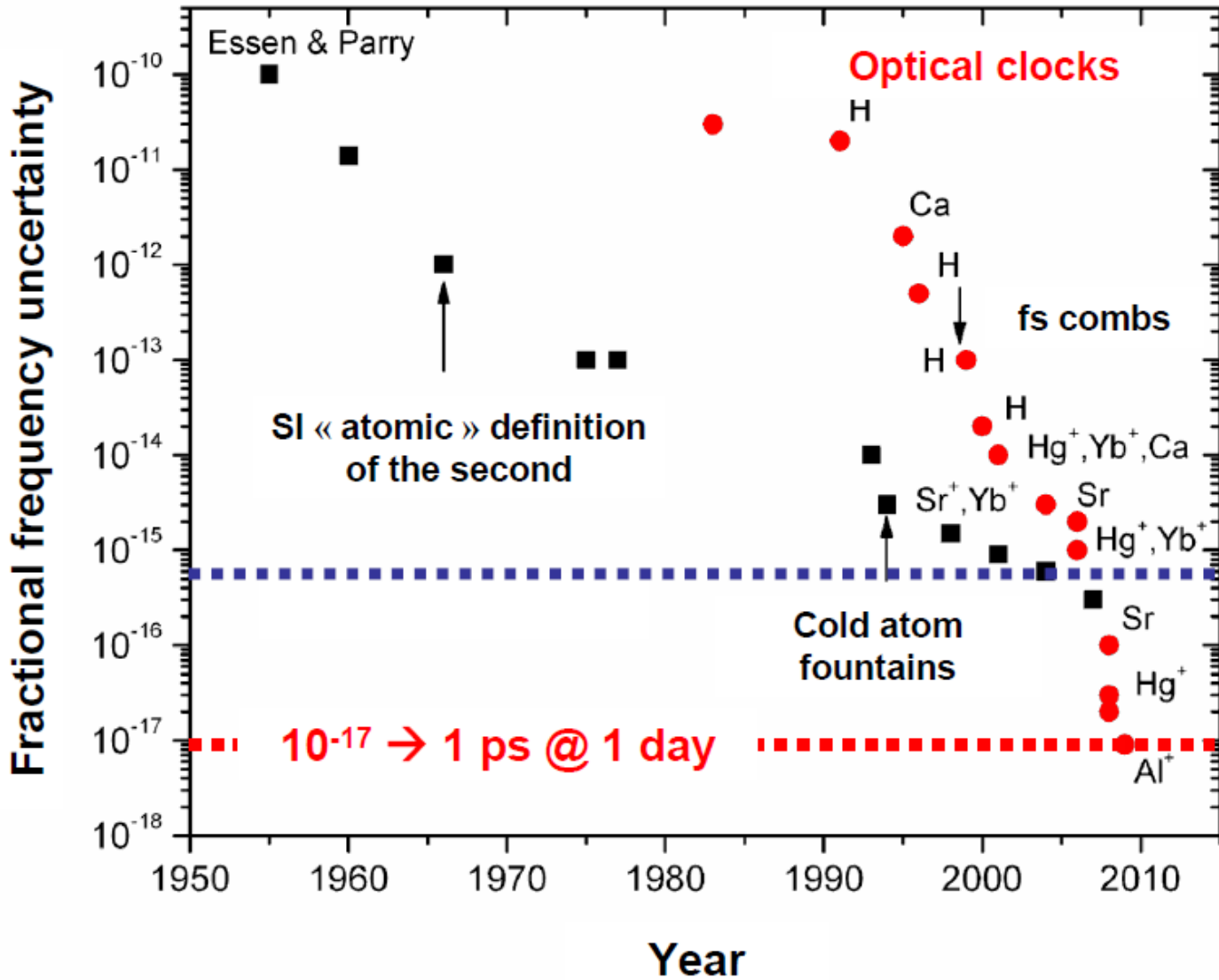
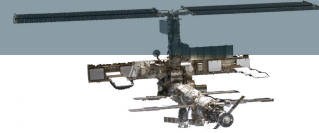
- Sr : Tokyo, JILA, SYRTE, PTB
- Yb : NIST, NMIJ, INRIM
- Hg : SYRTE

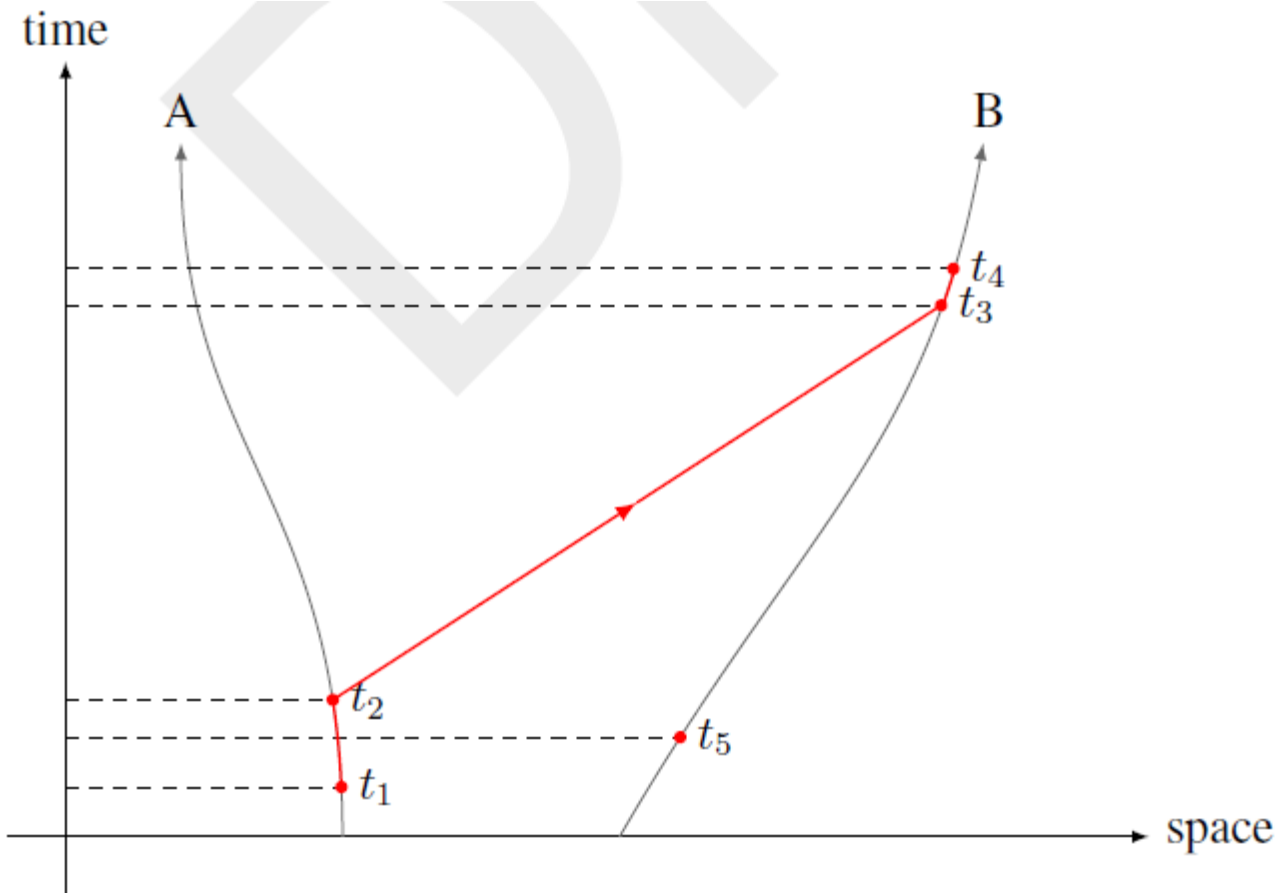
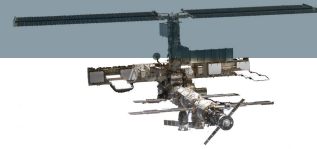






- Optical clocks have reached an accuracy better than the Cs fountain clocks
- They could be used as secondary representations of the second, to help characterizing the Primary Frequency Standards
- Their uncertainty cannot be better than the one of Cs fountains
- Possible future redefinition of the second
 - Period of preparation and study of different standards
 - Need to compare these secondary representations of the second at a challenging level





$$\begin{aligned} \Delta\tau^B (\tau^B(t_4)) &= \tau^B(t_5) - \tau^B(t_4) \\ &= \tau^A(t_1) - \tau^B(t_4) \end{aligned}$$



■ Current long distance global comparison methods

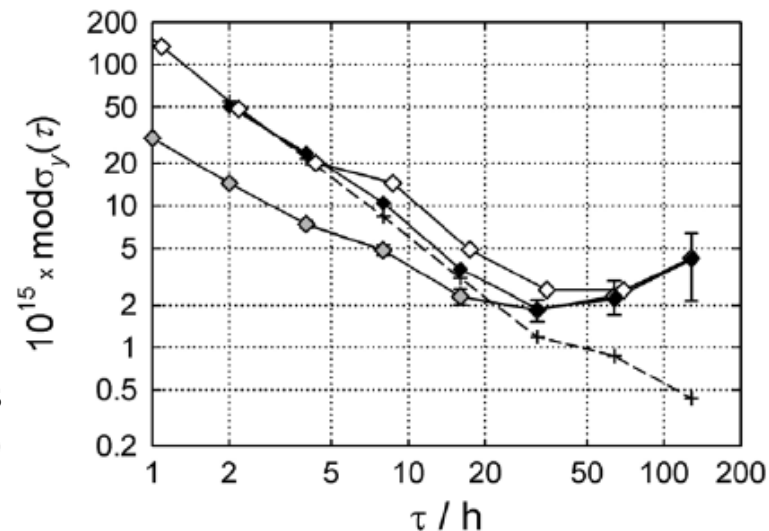
- GNSS-based techniques
- TWSTFT (using telecomms satellites)

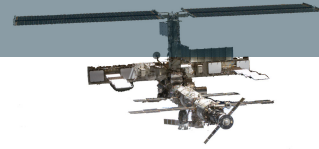
■ Performances

- Frequency comparison stability :
 $< 10^{-15}$ @ 1 day
- Proportional to $1/\tau$
- $\rightarrow \sim 10$ days to compare 2 fountains

! 2-3 order of magnitudes improvement needed !

IEN-OP comparison with 3 techniques
(GPS code, GPS phase TWSTFT)
(Bauch et al., Metrologia 2006)



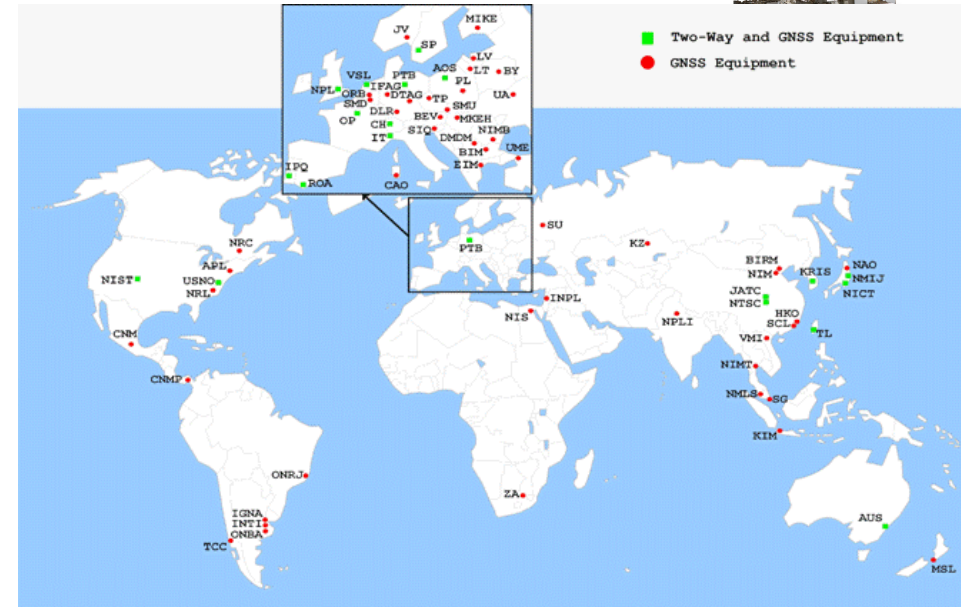


- 100-1000 km links demonstrated, e.g. Braunschweig-Munich 900 km
- $\sim 10^{-19}$ stability @ 1 day
- **Continental scales only**
- Intensive development going on
- Europe-wide network project NEAT-TF
- Fibre costs : using existing fibres dedicated to research



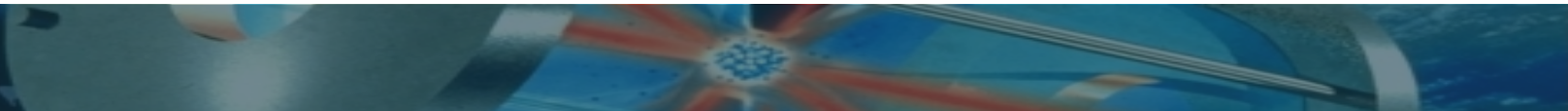


- T2L2 : Time Transfer by Laser Link
 - Launched on Jason-2 in 2008
 - Results consistent with objective of $< 10^{-16}$ @ 1 day
- ACES mission – Atomic Clock Ensemble in Space (2015)
 - Micro Wave Link (MWL)
 - Expected stability $\sim 10^{-16}$ @ 1 day and $< 10^{-16}$ @ 10 day
 - Always visible
- STE-QUEST mission – Space-Time Explorer and QUantum Equivalence principle teST (2022)
 - Coherent laser link
- Galileo Evolution : GNSS+ & ADVISE
 - See talk by F. Amarillo Fernandez tomorrow
 - Inter-satellite links in Galileo aiming at 1 cm accuracy on orbit determination and $< \text{ns}$ accuracy on synchronization
 - Implication on ground clock frequency comparison ???



- Useful timescale (coordinate timescale) :
 - Realize the unit (=second)
 - Continuous time reference
- Reference : Temps Atomique International (TAI) : timescale calculated by BIPM using ~250 clocks in more than 50 countries → frequency stability, robustness, accuracy, ...

- TAI unit (the second) is corrected by steering the frequency to the one of Caesium fountains (Primary Frequency Standards)
- TAI is independant from Earth rotation : $UTC = TAI - 34 \text{ secondes}$
- Clocks participating to TAI are compared with satellite time transfer



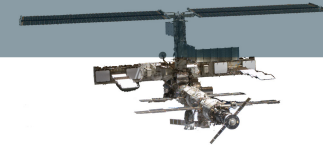


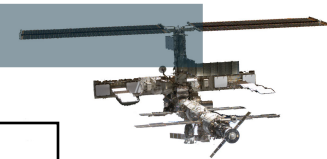
Table 1. Effects on syntonization with respect to TCG of clocks on the Earth's surface; Orders of magnitude and uncertainties of the corrections

Effect	Order of magnitude	Uncertainty
Earth's gravitational potential	7×10^{-10}	10^{-17}
Centrifugal potential ($v^2/2/c^2$)	1×10^{-12}	$< 10^{-18}$
Volcanic and coseismic (highly localised)	$< 10^{-16}$?
External masses (Moon, Sun)	10^{-17}	$< 10^{-18}$
Solid Earth tides	10^{-17}	$< 10^{-18}$
Ocean tides	10^{-17}	$< 10^{-18}$

P. Wolf and G. Petit, A&A (1995)

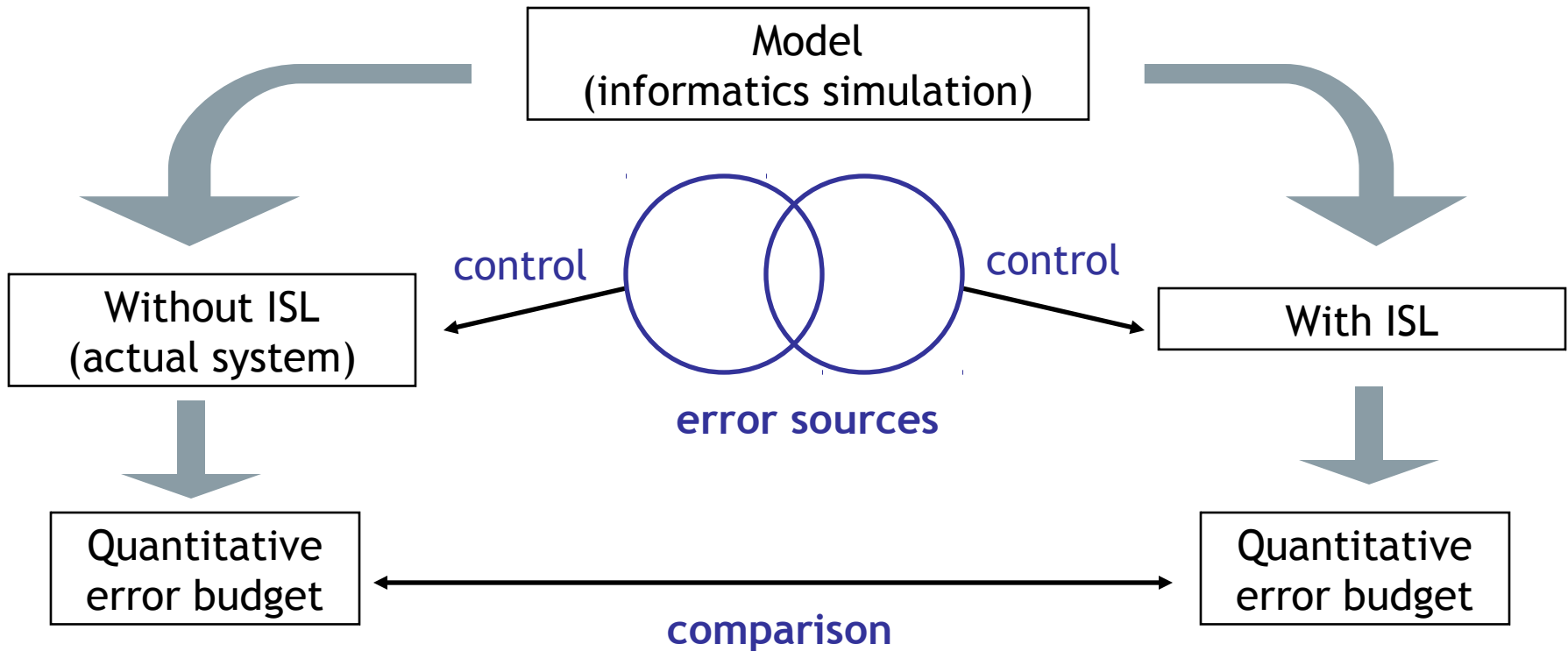
→ Realize a timescale in space ???

- Useful if you can achieve high accuracy : gravimetry
 - Use interconnected clocks (eg in GNSS) and correct them with a priori knowledge on their dynamics → ABC system
 - Put optical clocks in space



Quantitative study of spatio-temporal positioning

- Real time positioning
- High precision positioning
- Temporal positioning
- Earth-based user (stationary or moving)
- Space-based user (LEO satellite)





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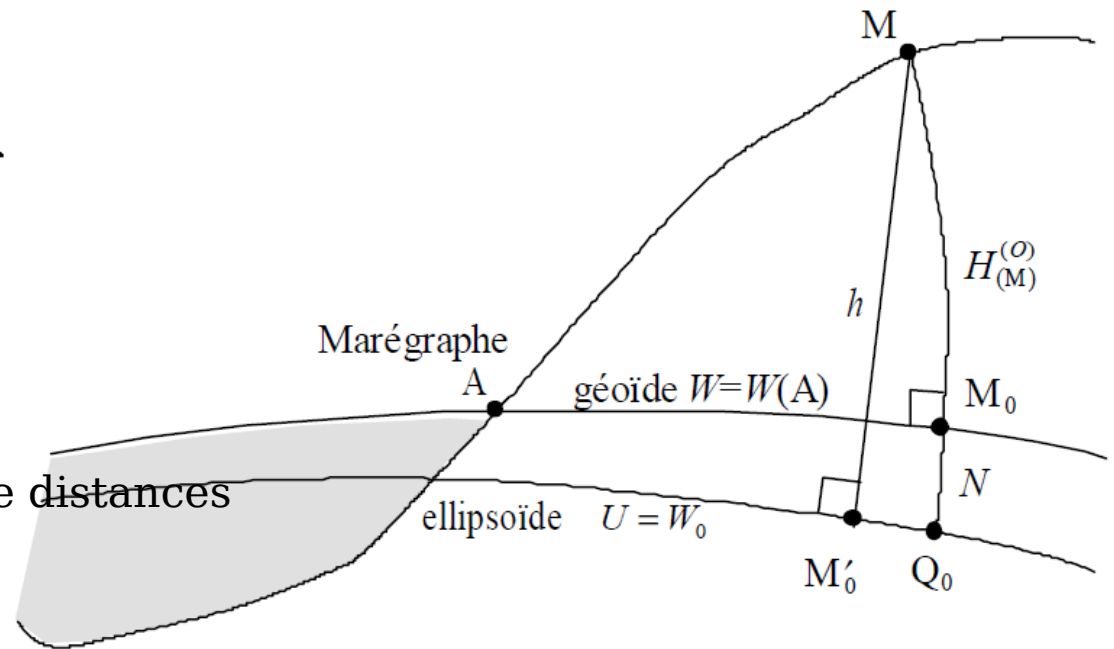
- Ellipsoid : terrestrial reference surface, purely geometrical
- Geoid : equipotential surface « nearest » to the mean sea level
- $N < 100$ m

- Geopotential number :
$$C_M = W_0 - W_M = - \int_P^M dW = \int_P^M g dn$$
 - Gravimeter, geometrical levelling

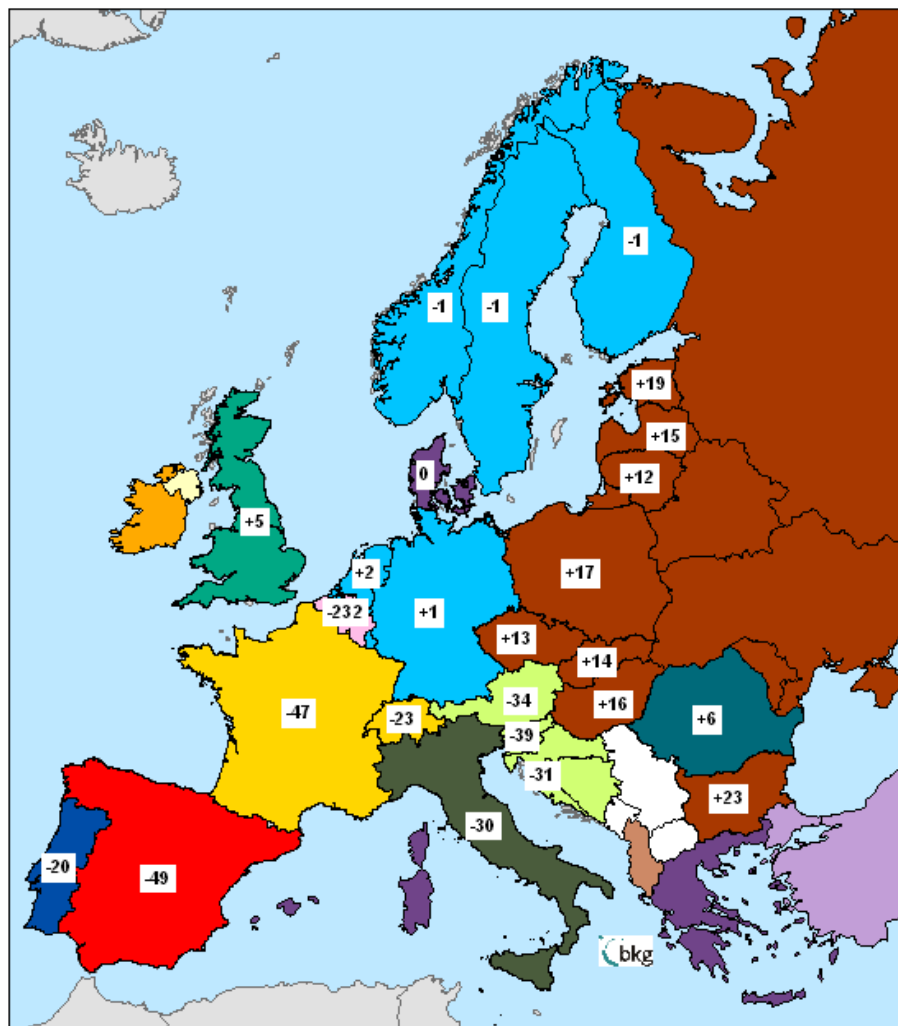
- Height : dynamic quantity that reflect the distance from the geoid (realized with tide gauges)

- $$H_{(M)}^{(*)} = \frac{W(A) - W(M)}{\gamma^{(*)}(M)}$$

- Normal height
- Orthometric height
- Dynamic height
- Errors accumulate on large distances



D. Bouteloup lecture (IGN, 2002)



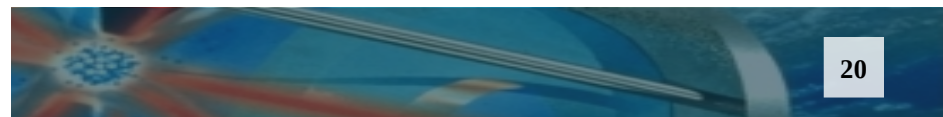
Reference tide gauges

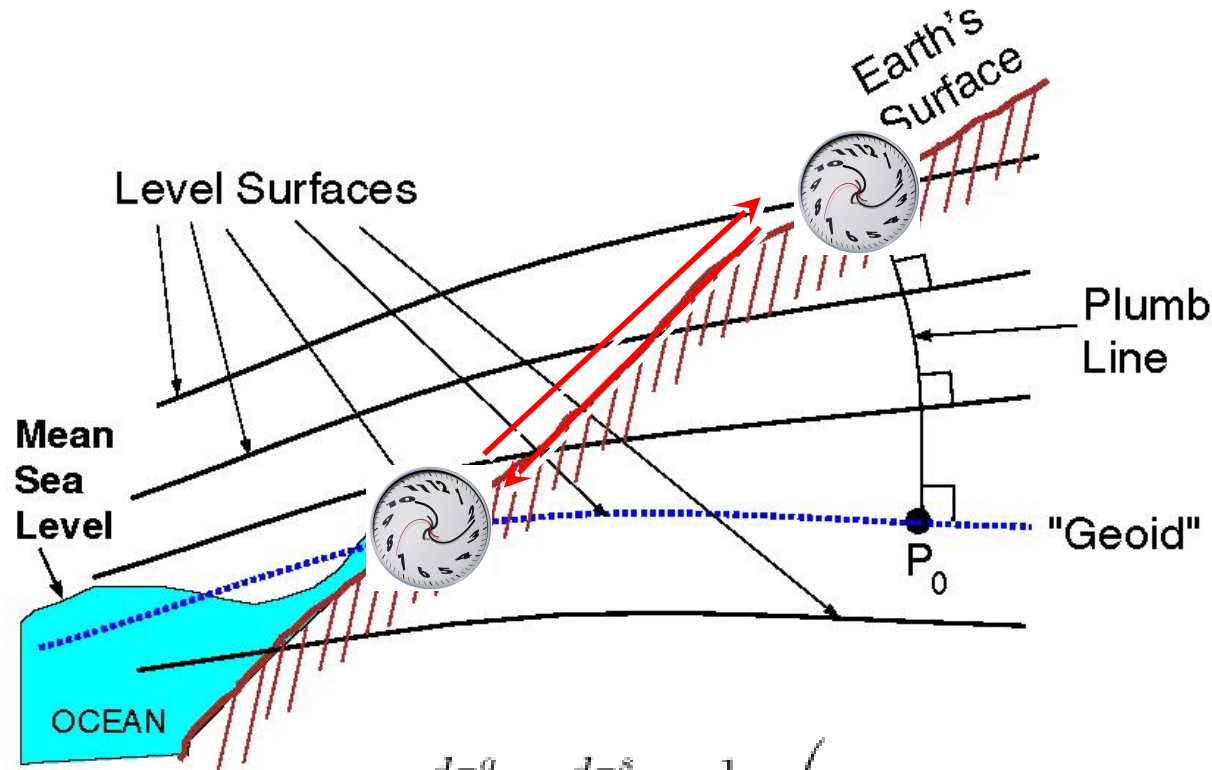
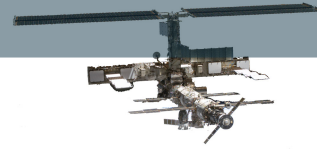
Alicante	Cascais	Kronstadt	Ostend
Amsterdam	Constanta	Malin Head	Trieste
Antalya	Durres	Marseilles	other
Belfast	Genoa	Newlyn	no information

« The national height reference systems of the European countries are tied to different reference points (tide gauges). Due to this fact and also as a result of the differences in the theoretical definition of heights, the reference heights may differ from each other by several dm, in maximum even up to 2 m. »

(BKG website)

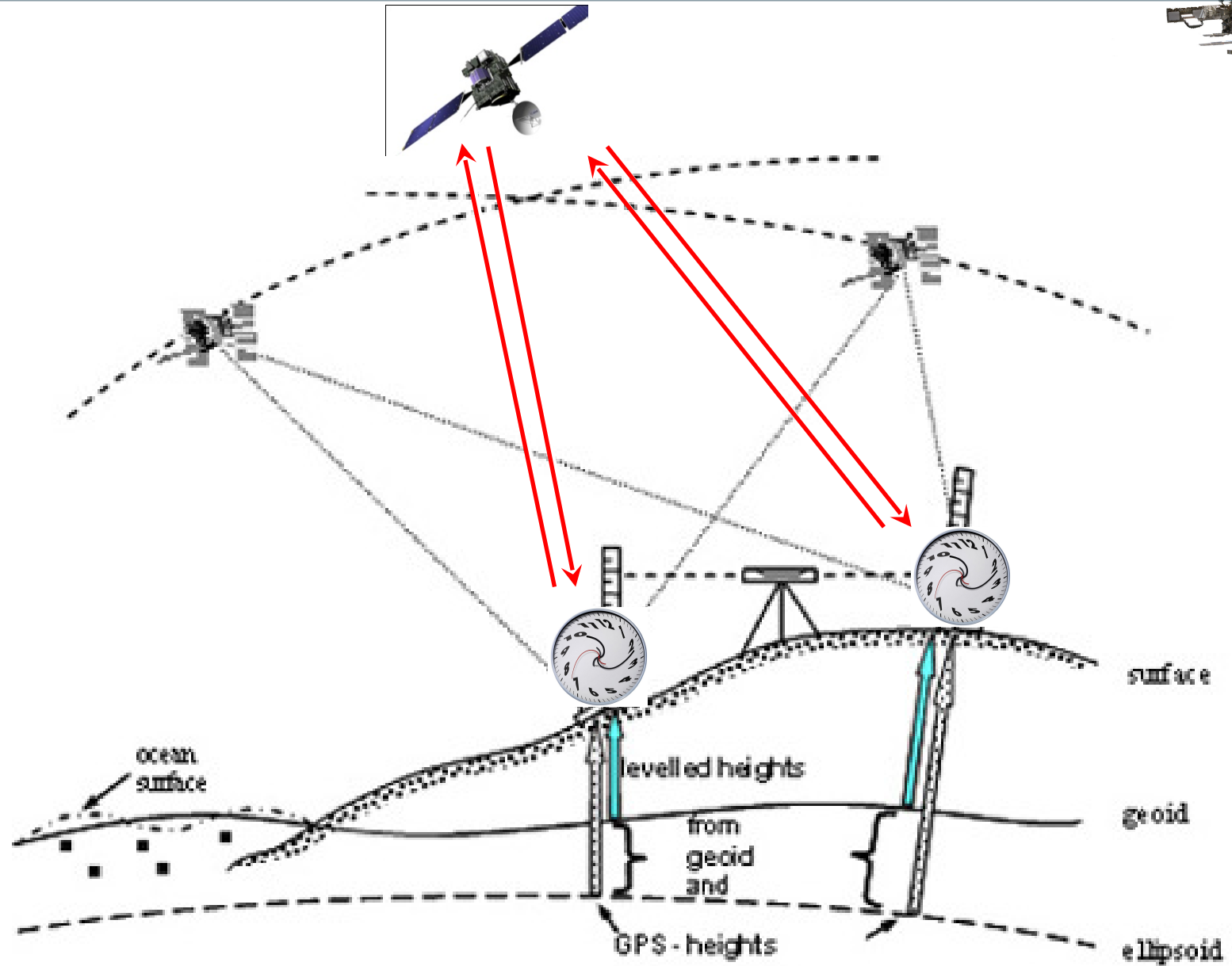
- One way out : global geoid model (satellite + ground gravimetry) + GPS measurements (~ 10 cm accuracy)





$$\frac{d\tau^g}{dt} - \frac{d\tau^s}{dt} = \frac{1}{c^2} \cdot \left(U(t, \vec{x}_s) - U(t, \vec{x}_g) + \frac{v_s^2(t)}{2} - \frac{v_g^2(t)}{2} \right) + O(c^{-4})$$

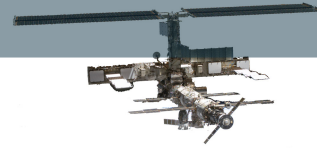
Three methods to determine potential differences





International timescales with optical clocks

- To make a complete evaluation of all relativistic effects influencing time and frequency comparisons at the 10^{-18} level of accuracy.
- To establish a connection to geodetic models in order to describe the variation of the clocks due to changes in the gravity potential, e.g. related to tidal effects
- To investigate the possibility of using continuously operated transportable optical clocks for the comparison of remote clocks
- To carry out a proof-of-principle experiment using optical clocks to measure gravity potential differences
- To perform an analysis of the complete frequency ratio measurement matrix derived from the optical clock comparison programme
- To consider how the output from this analysis could be used to derive steering corrections to UTC(k) timescales



- Reference in space : syntonization to Terrestrial Time
 - Ground clocks measure the « global » potential

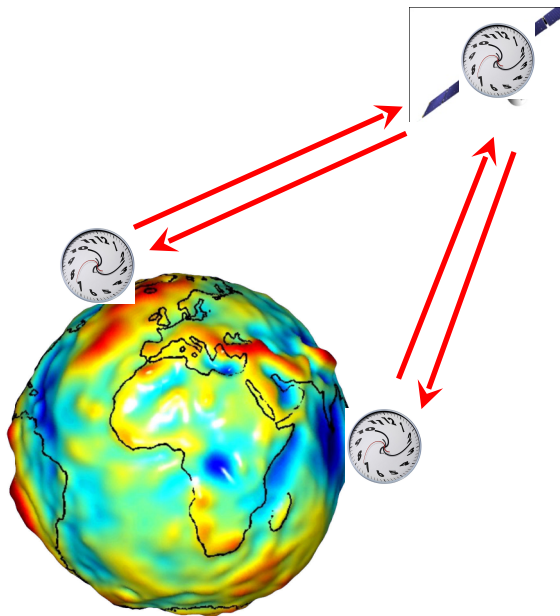


Table 2. Effects on syntonization with respect to TCG of clocks on board terrestrial satellites; Orders of magnitude and uncertainties of the corrections where h represents the altitude of the satellite

Effect	Order of magnitude	Uncertainty
Earth's gravitational potential	$< 6 \times 10^{-10}$	Few 10^{-18} (GEM-T3) $< 10^{-18}$ at $h > 4000$ km Few 10^{-18} (5 cm orbit uncertainty) $< 10^{-18}$ at $h > 10\,000$ km $< 10^{-18}$ at $h > 10\,000$ km
2nd order Doppler ($v^2/2c^2$)	$< 3 \times 10^{-10}$	
External masses :		
Moon	4×10^{-13}	} $< 10^{-18}$
Sun	4×10^{-14}	
Venus	6×10^{-18}	
Solid Earth tides	} 10^{-18} (at low altitudes)	} $< 10^{-18}$
Ocean tides		
Polar motion		
Atmospheric pressure		

- Relativistic geoid : surfaces of constant clock rates

$$W_g = W_E + \frac{1}{2} V^2 - \frac{W_E^2}{2c^2} - \frac{4W_E^b V^b}{c^2} + \frac{3W_E V^2}{2c^2} + \frac{V^4}{8c^2}$$

J. Muller et al, J. Geod. 82 (2008)



Realize the primary reference frame in space

- Time and frequency metrology
 - Better time unit (s)
 - Desynchronisation over large distance
 - Precise frequency dissemination
- Navigation
 - Ultra precise tracking of spacecrafts
 - Improve satellite navigation systems
- Geodesy
 - Earth's gravitational potential determination
- Fundamental physics
 - Test of general relativity
 - Link to the ICRF

