Relativistic positioning systems:

perspectives and prospects

Bartolomé Coll

Sistemas de posicionamiento relativistas
Departamento de astronomía y astrofísica
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Relativistic Positioning Systems and their Scientific Applications - Brdo, 19-21 September 2012

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Relativistic positioning systems are interesting ***technical objects*** for applications around the Earth and in the Solar system. But above all else, they are basic ***scientific objects*** allowing developing relativity from their ***own*** concepts. Past and future features of relativistic positioning systems, with special attention to the developments that they suggest for an ***epistemic relativity*** (experimental approach to relativistic physics), are analyzed. This includes ***relativistic stereometry,*** which, together with relativistic positioning systems, allow to perform the general relativistic notion of ***laboratory*** (space-time region able to receive experiments of finite size).

Relativistic Positioning Systems and their Scientific Applications - Brdo, 19-21 September 2012

Part 1.

What *relativistic positioning systems* learn to us

Part 2.

What relativistic positioning systems aim: an *epistemic relativity*

Part 3.

What epistemic relativity needs: a *relativistic stereometry*

Part 4.

What about the *mathematics* of relativity

Part 1
What *relativistic positioning systems* learn to us

♦ the genesis of relativistic positioning systems:
obstructions due to *prejudices*

♦♦ an example of an *extended prejudice*:
the OPERA experiment

♦♦♦ relativistic and classical positioning systems:
their *differences*

♦♦♦♦ the *main* relativistic positioning systems

Part 1
What *relativistic positioning systems* learn to us

♦ the genesis of relativistic positioning systems:
obstructions due to prejudices

TB Bahder

Navigation in curved space-time

Am. J. Phys. **69** (3), 2001, pp 315-321.

A covariant and invariant theory of navigation in curved space-time with respect to electromagnetic beacons is written in terms of J. L. Synge's two-point invariant world function. Explicit equations are given for navigation in space-time in the vicinity of the Earth in Schwarzschild coordinates and in rotating coordinates. The restricted problem of determining an observer's coordinate time when their spatial position is known is also considered.

B Coll

Elements for a theory of relativistic coordinate systems. Formal and physical aspects

Proc. Spanish Relativity Meeting ERE-2000, Reference Frames and Gravitomagnetism, Valladolid, September 6-9, 2000, World Scientific, pp 53–65 (also http://coll.cc)

Theorem: *In 2D Minkowski space-time, let S_1 and S_2 be two geodesic satellites emitting their proper times. Let s_1 and s_2 be respectively their values measured by an user U , and s'_1 and s'_2 the values respectively measured by S_2 and S_1 at the instants s_2 and s_1 . Then, the space-time metric is given by*

$$ds^2 = \sqrt{\frac{s_1\, s_2}{s'_1\, s'_2}}\; ds_1\, ds_2 \;.$$

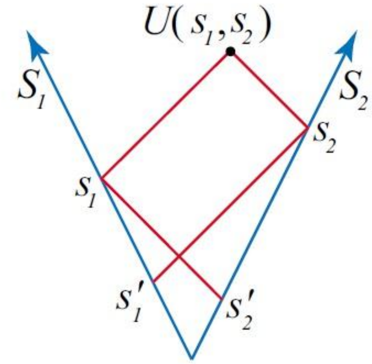


Figure 3: In 2D Minkowski space-time, the coordinates of an event and that of the two satellites allow to completely know the metric at the event.

C Rovelli

GPS observables in general relativity

arXiv: gr-qc/0110003 v1, 1 Oct 2001

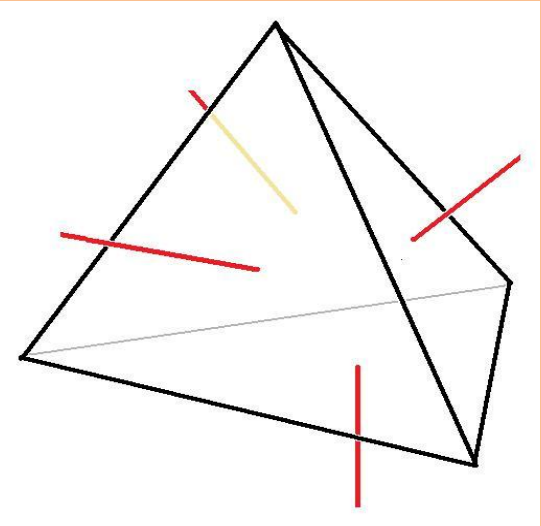
I present a complete set of gauge invariant observables, in the context of general relativity coupled with a minimal amount of realistic matter (four particles). These observables have a straightforward and realistic physical interpretation. In fact, the technology to measure them is realized by the Global Positioning System: they are defined by the physical reference system determined by GPS readings. The components of the metric tensor in this physical reference system are gauge invariant quantities and, remarkably, their evolution equations are local.

B Coll

Light coordinates in Relativity

(English translation)

Proc. Spanish Relativity Meeting 85, Ed. Ser. Pub. Univ. Barcelona, 1985, pp 29-38.



B Coll, JA Morales

Symmetric frames on Lorentzian spaces

J. Math. Phys. **32** (9), 1991, pp 2450-2455.

In the four-dimensional case this gives seven different types of symmetric frames. In them, the Minkowski metric diag.$(1,-1,-1,-1)$ adopts the form

$$\beta\begin{pmatrix}1-\kappa & 1 & 1 & 1\\ 1 & 1-\kappa & 1 & 1\\ 1 & 1 & 1-\kappa & 1\\ 1 & 1 & 1 & 1-\kappa\end{pmatrix},$$

where $\beta>0$ and $0<\kappa<4$. The frames corresponding to $\kappa=1$, $\kappa=2$, and $\kappa=3$ have null vectors, null planes, or null hyperplanes, respectively. Note that this classification is finer than the Derrick's[4] one, five of his ten types being obtained by adding the time orientation.

B Coll, JA Morales

199 causal classes of space-time frames

Int. Jour. of Theo. Phys. **31** (6), 1992, pp 1045-1062.

The 199 Causal Classes of Space-time Frames

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t , l , e / t , l , e / T , L , E are respectively time-like , light-like , space-like coordinate: hypersurfaces or covectors / lines or tangent vectors / surfaces or planes

The 6 surfaces or planes of every class $X_1X_2X_3X_4X_5X_6$ are generated respectively by the 4 hypersurfaces or covectors x_1, x_2, x_3, x_4 in the following order

$X_1 = x_1 \wedge x_2$ $X_2 = x_1 \wedge x_3$ $X_3 = x_1 \wedge x_4$

$X_4 = x_2 \wedge x_3$ $X_5 = x_2 \wedge x_4$ $X_6 = x_3 \wedge x_4$

B. Coll and J.A. Morales, The 199 Causal Classes of Space-time Frames, *Internat. J. Theo. Phys.*, **31**, 6, p 1045-62 (1992)

Every one of these papers has contributed to the removal of at least one of the following three prejudices:

"a physical frame must involve necessarily space-like synchronizations"

"no real frame of four null vectors exists in relativity"

"coordinate systems have no physical meaning"

"a physical frame must involve necessarily space-like synchronizations"

- is more a feeling-based prejudice than an error-based one,
- is related to the old prejudice, current some decades ago in metrology, that a standard of distance must be matter-based and not clock-based,
- is a remainder of the feeling that an extended instantaneous space is physically meaningful.

This prejudice was opposed to our works:

- Light coordinates in Relativity,
- 199 causal classes of space-time frames,
- Elements for a theory of relativistic coordinate systems,
Formal and physical aspects.

"no real frame of four null vectors exists in relativity"

- is an error-based prejudice,
- due to the abundance of works in the well-known Newman-Penrose formalism of "null" tetrads,
- the error is to apply **unconsciously,** to any set of four null vectors, the **orthogonality condition** of Newman-Penrose "null" tetrads.

This prejudice was opposed to our works:

- Light coordinates in Relativity,
- 199 causal classes of space-time frames,

"coordinate systems have no physical meaning"

- is an error-based prejudice,
- due to an incorrect statement of the *principle of general covariance*

--ooOoo--

It states that:

the laws of physics are* invariant *by the choice of coordinate systems

but during dozens and dozens of years, this statement has slid to the form

the laws of physics are* independent *of the choice of coordinate systems

This prejudice was opposed to our works:

- Light coordinates in Relativity,
- 199 causal classes of space-time frames,

- Due to it, a hierarchical superior of the CNRS department forbade me to send for publication the content of ***Light coordinates in Relativity***,

- The paper ***199 Causal Classes of Space-Time Frames*** was refused by 3 journals and criticized by colleagues on the basis, in part, of the three above mentioned prejudices.

The main obstructions to innovative research are the prejudices.
The own prejudices for its conception. Those of the peers for its diffusion.

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The own prejudices for its conception. Those of the peers for its diffusion.

Message 1[1]	Be very careful with the conditions under which every assertion is true.

____

[1]Specially to young people

The main obstructions to innovative research are the prejudices.
The own prejudices for its conception. Those of the peers for its diffusion.

Message 1[1]	Be very careful with the conditions under which every assertion is true.

Message 2[1]	Think in four dimensions.

[1]Specially to young people

Part 1
What *relativistic positioning systems* learn to us

♦♦ an example of an extended prejudice: the OPERA experiment

- In 2011, the **OPERA** experiment between the **CERN** (Geneva) and the **LNGS** (Gran Sasso) *mistakenly reported neutrinos appearing to travel faster than light.*[1]
- … these two sources of error *eliminated the faster-than-light results*.[1]
- In March 2012, the collocated ICARUS experiment … obtained agreement with the speed of light.[1]
- On June 8, 2012 CERN research director Sergio Bertolucci declared … that the speed of neutrinos is consistent with that of light.[1]
- On July 12, 2012 OPERA … found agreement of neutrino speed with the speed of light.[1]

___

[1] http://en.wikipedia.org (page modified on 9 August 2012)

- Prof Stephen Hawking … expressed doubts, saying: 'It is premature to comment on this. Further experiments and clarifications are needed'.[2]

- … scientists reported that they had clocked … neutrinos going faster than the speed of light, to the astonishment and vocal disbelief of most of the world's physicists, …[3]

- `It looks too big to be true´, Alvaro de Rujula, a CERN theorist, said at the time.[4]

Elwood H. Smith

Particles Faster Than the Speed of Light? Not So Fast, Some Say[4]

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[2] http://physicsforme.wordpress.com
[3] http://www.physicstoday.org
[4] http://www.nytimes.com

All these comments, and almost any others in different media,
show that directors of the experiment
but also the most part of the physicists
believe that

neutrinos travelling faster than the speed of light c between CERN and LNGS are inconsistent with standard relativity theory

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show that directors of the experiment
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Is this belief correct or is it a prejudice?

It is a prejudice!

Why?

Local theory

- A local theory is a theory whose statements and equations are local, i.e. valid in such small space-time regions that any physical quantity not mentioned in the statement or not appearing in the equations is constant.

- Roughly speaking, mathematically a physical theory is local if its formulation is infinitesimal (relates physical fields and their space-time variations at every event).

Relativity

- Relativity theory,

  - by the concepts used in its construction,
  - by the principles on which it is founded,
  - by the domain of influence of its equations,
  - by the causal character of them,
  - by the tensor character with which it represents the physical quantities
  - by the concept itself of space-time that it proposes,
  - and because it gives no phenomenological theory for the construction of its current (energy tensor), but supposes it can be obtained by means of classical balances,

is a local theory in all its constituents.

Light velocity

- Consequently, as all others general statements of the theory, the one that says

  the velocity of light c = 299 792 458 m / s is a physical limit,

  or any other equivalent version, is a **local** statement.

- Being local, the **velocity** cannot but be an **instantaneous velocity,** i.e. measured in a so small time interval that any physical quantity not implied by the concept of *velocity* is constant. It cannot be, in general, a **mean velocity**.

Einstein versus Newton

- In Newtonian theory, where time and space are absolute, if the instantaneous velocity of a particle remains constantly lesser than a value v during a finite interval, the mean velocity in this interval will also be lesser than v.

- But in relativity, where time and space are different at different events, a particle whose instantaneous velocity remains constantly lesser than a value v during a finite interval, may have a mean velocity lesser, equal or greater than v.

Einstein versus Newton

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- But in relativity, where time and space are different at different events, a particle whose instantaneous velocity remains constantly lesser than a value v during a finite interval, may have a mean velocity lesser, equal or greater than v.

In spite of the fact that this last property is very easy to see,
It is its erroneous Newtonian version that largely prevails

The OPERA experiment

- In the OPERA experiment it is measured the mean velocity of the neutrinos between the CERN and the LNGS, two events of *different* gravitational acceleration, traveling a region of a *non constant* gravitational field.

- It is then possible to show[1] that:

Relativity theory predicts a mean velocity GREATER than c for neutrinos travelling with instantaneous velocity c

[1]B. Allés, Phys. Rev. D 85, 047501 (2012).

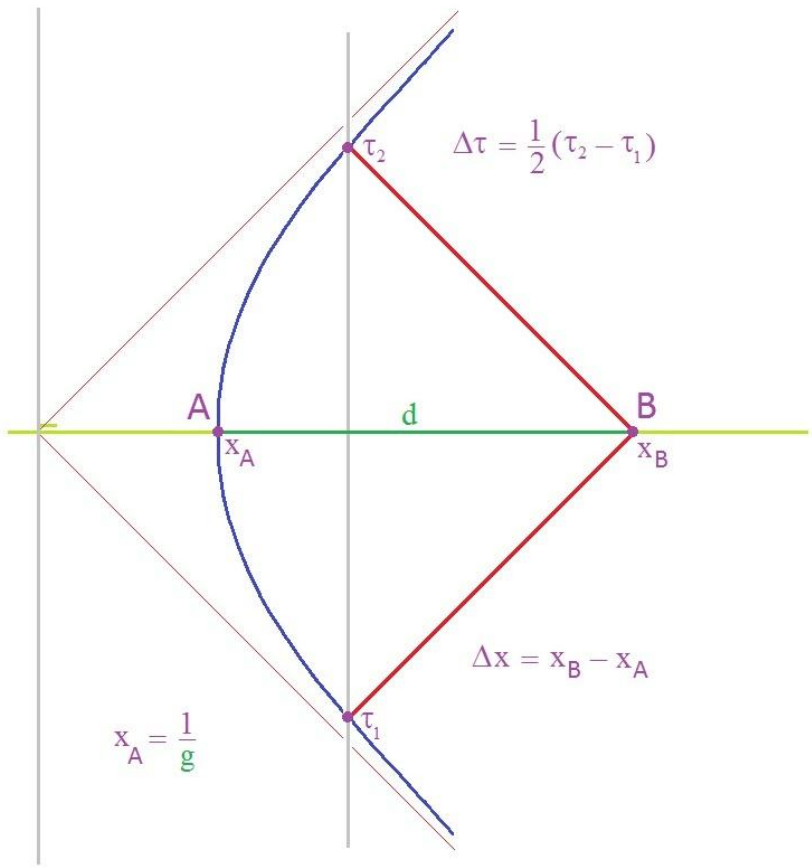
Balance of the OPERA experiment

- The two errors detected in the first evaluation of the OPERA experiment invalidate the quantitative result, *the numerical value,* too great, obtained for the mean velocity of the neutrinos.

- But the qualitative result that, between the CERN and the LNGS, neutrinos go at a mean velocity greater than the universal constant c remain true!

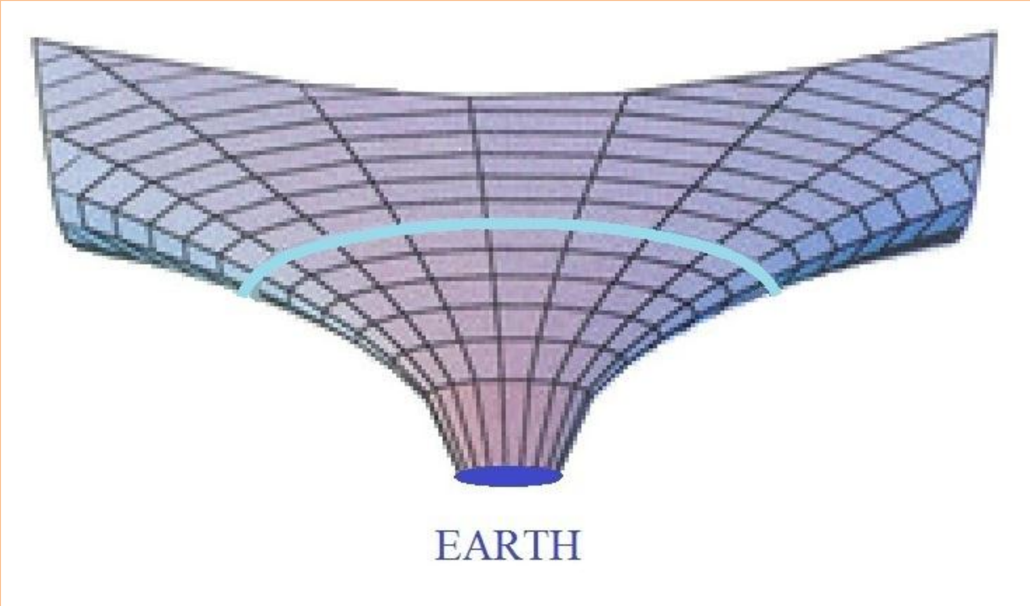
- The "consistence" or "agreement" of this mean velocity with the value of **c** established in subsequent evaluations is not but a measure of the degree of uncertainty of the experiment, because **what relativity predicts is a value greater than c.**

Example of mean velocity greater than c



$$v_m = c\,\frac{\frac{gd}{c^2}}{ln\left(\frac{gd}{c^2}+1\right)} \cong \frac{c}{1-\frac{1}{2}\frac{gd}{c^2}}$$

In a RGNSS, the aim of a relativistic positioning system is thus
to characterize the physics of the space-time region
between the constellation of satellites and
the Earth surface

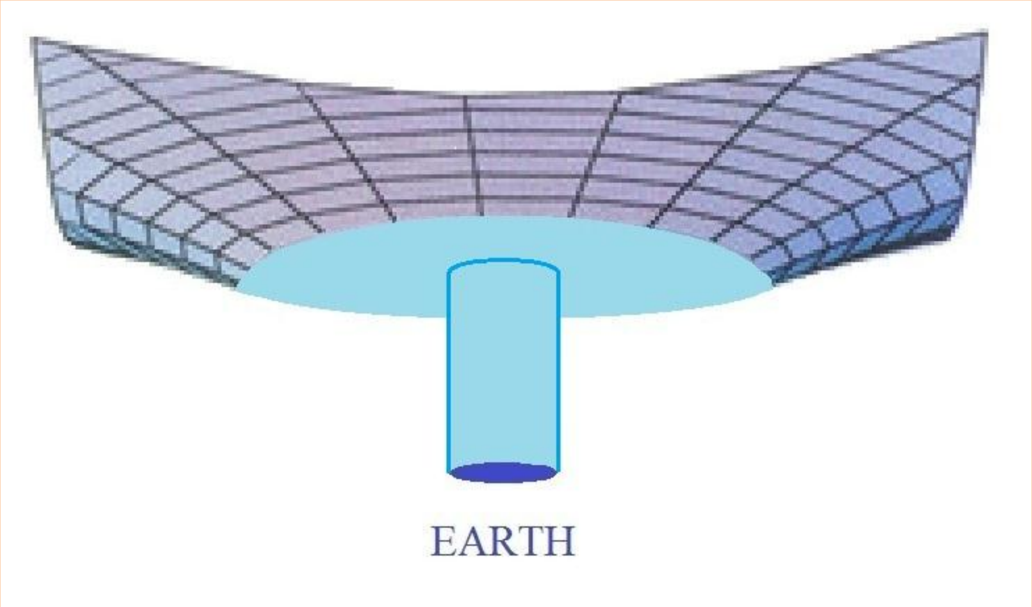


- The aim of a classical positioning system is:
    - to allow any user to know its position in time and space with respect to a Newtonian conventional choice of time scale (TAI) and distance.

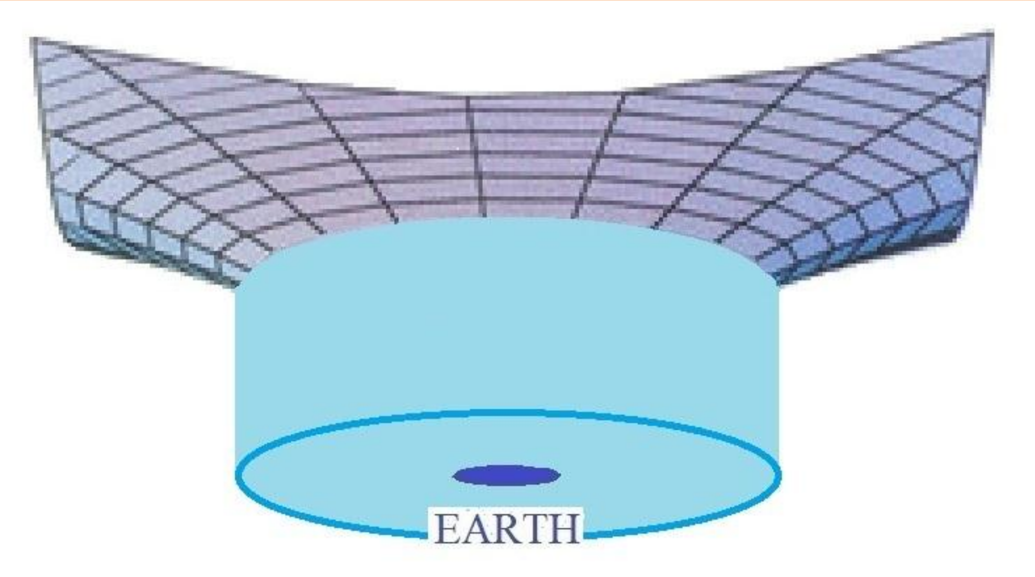
- TAI (or UTC) is a partially political, social and physical time obtained from weighted atomic clocks of all the states that contribute to its construction.

  TAI is a sort of mean proper time on a model of sea surface.

- To impose TAI not only all over the Earth surface, but also at any altitude is tantamount to a Newtonization of the space-time region between the Earth surface and the satellite constellation.



- Other Newtonizations of the same region are possible:



- Although for a smooth running of both systems, RGNSS and GNSS, the same information is needed, this information is obtained and used very differently.

  For example, in Ashby works on relativistic corrections to the GPS, these corrections are used "substractively" in order to make the Newtonization as precise as possible.

So that, in positioning, the third message is

Message 3[1]	Be conscientious of what you are doing: using relativity to better Newtonize GNSS, or using relativity to construct a relativistic positioning system

And to avoid confusion, I recommend:

[1] Specially to young people

So that, in positioning, the third message is

Message 3[1]	Be conscientious of what you are doing: using relativity to better Newtonize GNSS, or using relativity to construct a relativistic positioning system

And to avoid confusion, I recommend:

Message 4[1]	In working with the structure of relativistic positioning systems forget completely the classical ones

[1] Specially to young people

Part 1
What *relativistic positioning systems* learn to us

♦ ♦ ♦ ♦ The main relativistic positioning systems

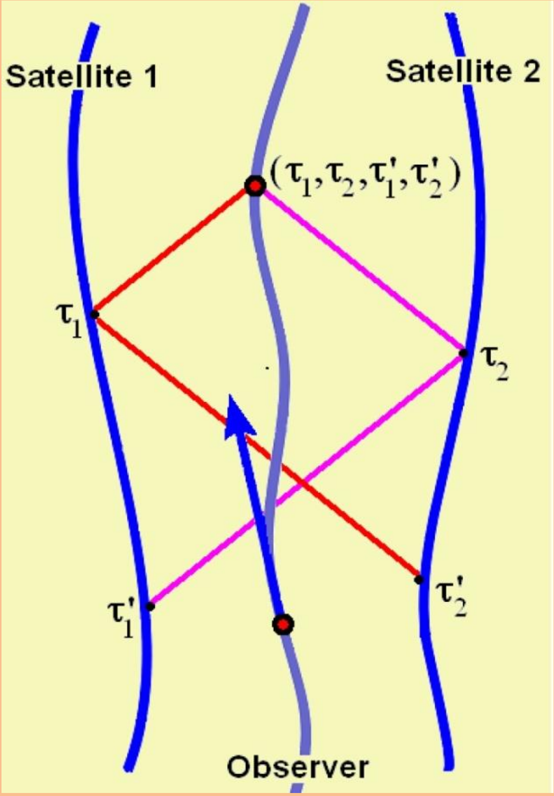
- A ***location system*** is a physical realization of a coordinate system.
- We know that there exist two important classes of **location systems:** ***reference systems*** and ***positioning systems.***
- Positioning systems are restricted to be ***generic***, ***free*** and ***immediate*** *So that they are better location systems* than reference systems.

- A**uto-locating positioning systems** are those that broadcast their proper emission coordinates.

- Auto-locating systems allow any user to *draw* the trajectories of the satellites in emission coordinates.

- But the user still does not know *how to draw* emission coordinates in the space time in which he is living.

- To be able to do this, the **coordinate data** broadcast by the auto- locating system has to be completed with:



  - **dynamical data** of the satellites (acceleration, gradiometry),

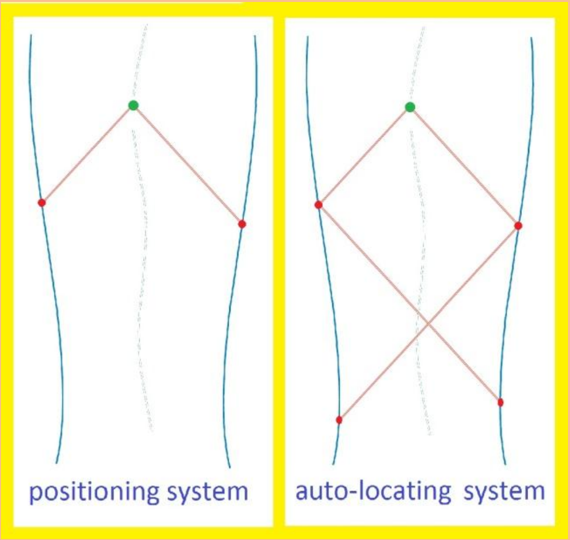
  - **observational data** from them (e.g. position of quasars)

- Auto-locating systems broadcasting these **autonomous data** are called:

autonomous positioning systems.

- On the other hand, simple positioning systems are manifestly **incomplete** and **incoherent** because they need to be referred to another, non immediate, location system
(a reference system).

  In this sense, they are *hybrid* between *reference systems* and *autonomous positioning systems.*



the best *location systems* are the *autonomous positioning systems*

the best *location systems* are the *autonomous positioning systems*

Message 5[1]	Think mainly in **autonomous positioning systems**. They are the challenge.

[1] Specially to young people

Autonomous positioning systems were already introduced in 2000,
in my first paper on relativistic positioning systems:

Elements for a theory of relativistic coordinate systems.
Formal and physical aspects

Proc. Spanish Relativity Meeting ERE-2000, Reference Frames and Gravitomagnetism, Valladolid, September 6-9, 2000, World Scientific, pp 53–65 (also http://coll.cc)

Theorem: *In 2D Minkowski space-time, let S_1 and S_2 be two geodesic satellites emitting their proper times. Let s_1 and s_2 be respectively their values measured by an user U, and s_1' and s_2' the values respectively measured by S_2 and S_1 at the instants s_2 and s_1. Then, the space-time metric is given by*

$$ds^2 = \sqrt{\frac{s_1\, s_2}{s_1'\, s_2'}}\; ds_1\, ds_2 \, .$$

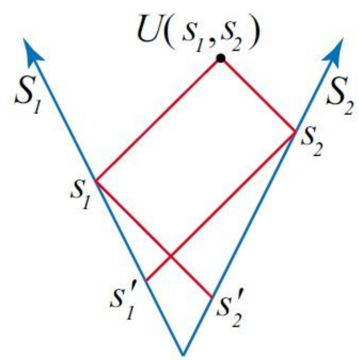


Figure 3: In 2D Minkowski space-time, the coordinates of an event and that of the two satellites allow to completely know the metric at the event.

Part 2
What *relativistic positioning systems* aim: an epistemic relativity

♦ My confession

♦♦ Epistemic relativity

♣ **C**oncept

♣♣ **O**bjective

Part 2
What *relativistic positioning systems* aim: an epistemic relativity

♦ My confession:

- I don´t agree with the development of relativity theory during its century of existence.
- Relativity theory is a physical theory of the gravitational field, but it also is a theory of the space-time frame in which all physical phenomena take place.
- Both constituents improve their homologues in Newtonian theory.

- For these reasons, I think that:

  - simple experimental verifications from time to time, as it is the practice, are largely insufficient for a physical theory,

  - as an improved theory of the space-time frame, ***any*** physical experiment ought to be conceptually described in relativity, regardless of its quantitative evaluation, for which in many cases Newtonian calculations could suffice.

  - as an improved theory of the gravitational field, relativity ought to propose experiences and methods of measurement of general gravitational fields (four-dimensional metric), which, up to now, are conspicuous by their absence.

- In short:

relativity needs to develop a proper experimental approach to the physical world

And I believe that we already have the conceptual basic ingredients for it.

- Now, for this purpose, we need to make more precise the idea of an:

relativistic experimental approach.

♦ ♦ Epistemic relativity

♣ Concept

=> In relativity, a good deal of the scientific publications:

* analyze physical and geometrical properties of the space-time,

but

* dont integrate the physicist as a part of it,

and

* forget implicitly that
  - o information is energy
  - o neither the density of energy, nor its velocity of propagation can be infinite in relativity

=> Many of these properties of the space-time,

* may be analyzed by a geometer on his desk,

* but, to be known by an experimental physicist, would require the qualities of an

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omniscient god !

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* may be analyzed by a geometer on his desk,
* but, to be known by an experimental physicist, would require the qualities of an

omniscient god !

=> For these reasons, we characterize these publications as belonging to

ontic relativity

(from Greek `ontos', *being*, with the meaning of `what is´ as opposed to `who sees it´)

=> The works in relativity that:

* *integrate* the physicist as an element of the problem considered,
* concern physical properties that the physicist can *measure*,
* take into account explicitly *what information, when and where*, the physicist is able to know,

will be considered as characterizing

epistemic relativity

(from Greek `episteme', *knowledge*, with the meaning of `how we obtain it´)

♣♣ Objective

=> The *main* objective of epistemic relativity is to provide the physicist with the protocols and knowledge necessary to make relativistic gravimetry in its unknown space-time environment.

This is the *first* and *unavoidable* step to develop experimental relativity as the natural scientific approach to our physical world.

♣♣ Objective

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This is the *first* and *unavoidable* step to develop experimental relativity as the natural scientific approach to our physical world.

Message 6[1]	Be epistemic with relativistic positioning systems.

[1] Specially to young people

Part 3

What epistemic relativity needs?
- relativistic stereometry

♦ What is a laboratory?

♦♦ What is an observer?

♦♦♦ The aim of relativistic stereometry

♦♦♦♦ First theorems in relativistic stereometry

Part 3

What epistemic relativity needs?
- relativistic stereometry

♦ What is a laboratory?

- It is evident that relativistic positioning systems are the *first* ingredient of *epistemic relativity*:
    - In the space-time, the adequacy of a *mathematical* model and the *physical* system that it describes needs of a univocal correspondence between them,
    - And because in the differentiable manifold of the mathematical model points are *identified* by their coordinates, we need to know how to construct physically a *coordinate system* (location system).

- What other else do we generically need in epistemic relativity?

  - In fact, what we need is to transform *the space-time region of physical interest* in a **laboratory**.

- But, what is a **laboratory** *of finite dimension* in relativity?

  - In fact, any **laboratory**, *regardless of the specificity of its measurement devices*, has to provide us with:

    - a precise *localization* of the significant parts of the physical system, and

    - a precise *description* of its intrinsic physical properties.

  - Similarly to localization, which is obtained with a system of four clocks (relativistic positioning system), the precise description of the **intrinsic properties** of a system has to be obtained with a **system of four (relativistic) observers**.

such a *system of four* ***observers*** is called a

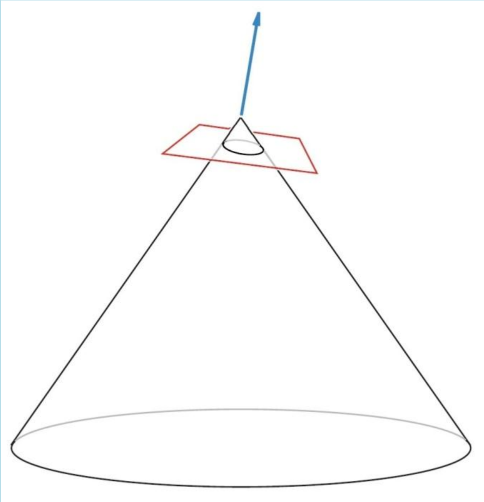
stereometric system

A finite laboratory in relativity is a space-time region endowed with
- a relativistic positioning system
and
- a relativistic stereometric system

♦♦ What is an observer?

- An observer is an eye able to *record* and to *analyze* the input.

- A (relativistic) eye at an event is a *differential object*, determined by its unit velocity, that projects the past light cone on its *celestial sphere*.

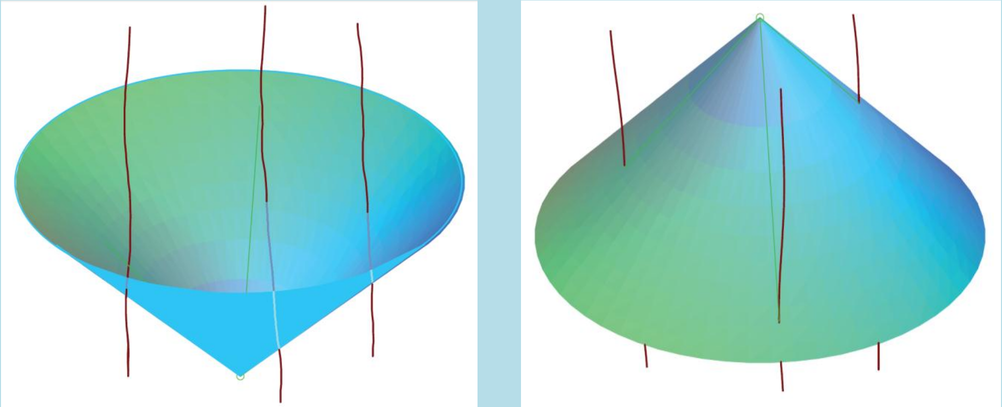


  There exists interesting papers on relativistic vision but:

  - they are very different in strategy, starting hypothesis and definition of \`eye´.
  - their number is very small,
  - none of them consider the invariants of the configuration studied.

♦ ♦ ♦ The aim of relativistic stereometry

relativistic stereometric systems are the causal duals of positioning systems



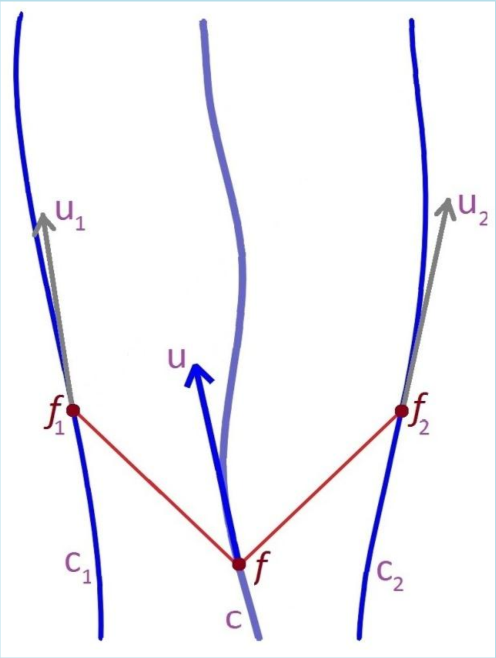
It is then clear that many of the properties of *one* of these systems may be transformed, by changing time orientation, in properties of the *other* system.

From the above concept of relativistic laboratory, it is clear that:

The **aim** of a relativistic stereometric system is to describe the **intrinsic properties** of physical systems

♦♦♦♦ First theorems in relativistic stereometry

- The intrinsic properties of a system being those which are observer-invariant, in relativistic stereometry we have to solve, starting from four observer-dependent perspectives, an **inverse problem**.



- In Minkowski two-dimensional space-time let C be the world-line of a colored point of a physical system, of *frequency* f.

  Let f_1 and f_2 be the Doppler *frequencies* received by the *stereometric system* and let v_{12} the module of the *relative velocity* of the *observers* C_1 and C_2 at the instants of reception of the signals f_1 and f_2.

Then we have:

Theorem 1: *In terms of the received frequencies* f_1 *and* f_2 *and of the relative velocity* v_{12} *of the system at the reception instants, the proper frequency* f *of the colored point* C *is given by*

$$f^2 = f_1 f_2 \frac{v_{12}}{\sqrt{1 - v_{12}^2}} \quad .$$

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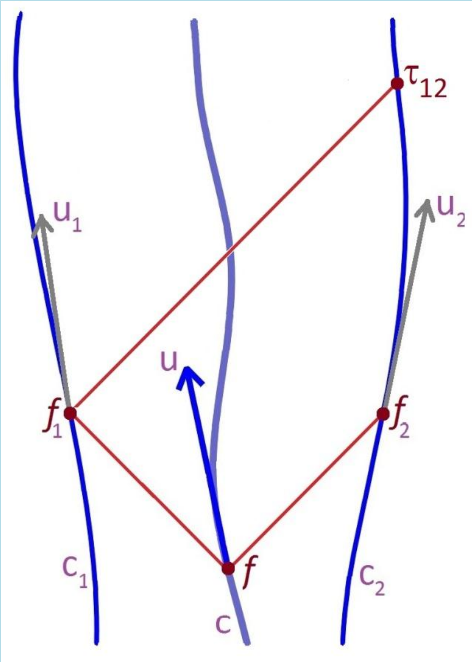
Theorem 2: *The relative velocities* v_1 *and* v_2 *of the point* C *with respect to the eyes* C_1 *and* C_2 *at the instants of reception of the signals* f_1 *and* f_2 *are given by:*

$$v_1 = \frac{f_1\sqrt{1 - v_{12}^2} - f_2 v_{12}}{f_1\sqrt{1 - v_{12}^2} + f_2 v_{12}} \quad , \quad v_2 = \frac{f_2\sqrt{1 - v_{12}^2} - f_1 v_{12}}{f_2\sqrt{1 - v_{12}^2} + f_1 v_{12}} \quad .$$

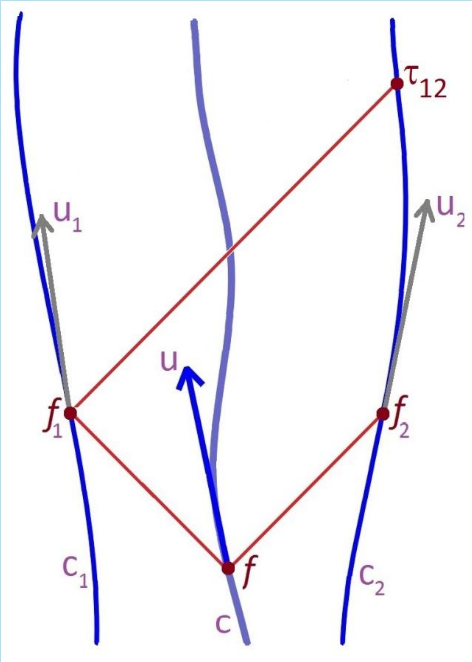


Are `epistemic´ these results?

The answer is `yes´:

Theorem 3: *The relative velocity* v_{12} *of the system at the reception instants, in terms of the rate* τ_{12} *of the proper time of the eye* C_1 *with respect to the proper time of the eye* C_2 *, is given by:*

$$v_{12} = \frac{\tau_{12}}{\sqrt{1+\tau_{12}^2}}$$



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Message 7[1]	**Be *epistemic* with relativistic positioning and stereometric systems.**

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[1]Specially to young people

Part 4

What about the mathematics of relativity?

♦ Insufficiency of differential geometry in relativity

♦ ♦ Finite-differential geometry

Part 4

What about the mathematics of relativity?

♦ Insufficiency of differential geometry methods in relativity

In front of the historical and conceptual need of differential geometry in relativity, there exist practical (epistemic!) insufficiencies of the differential methods in their present form:

- Classical fields of infinite range,
- Propagation of their perturbations at the velocity of light,
- Physically meaningful character of this propagation

The usual mathematical methods in relativity
are not well adapted to relativity

I think that it is an urgent task in relativity, for all of us, to try to find a:

- **finite-differential geometry**

♦♦ Finite-differential geometry

- The purpose of **finite-differential geometry** is to introduce **finite** and **interchangeable** versions of the basic ingredients of differential geometry:

  $\rightarrow$ metric g

  $\rightarrow$ connection Γ

  $\rightarrow$ curvature *Riem*

- The **distance function** $U(x,y)$, or its half-square $\Omega(x,y)$, the Synge´s **world-function**,

$$\Omega(x, y) \equiv \frac{1}{2} U(x, y)^2$$

are already finite versions of the metric g. As it is known:

$$\Omega(x, y) = \frac{1}{2} \int_0^1 g(\frac{\mathrm{d}\gamma}{\mathrm{d}\lambda}, \frac{\mathrm{d}\gamma}{\mathrm{d}\lambda}) \mathrm{d}\lambda \quad ,$$

And their fundamental equations are:

$$g^{\alpha\beta}\, \partial_\alpha \Omega\ \partial_\beta \Omega = 2\Omega \quad , \quad g^{ab}\, \partial_a \Omega\ \partial_b \Omega = 2\Omega \quad .$$

- Distance spaces are well known, but their link with differential geometry is not sufficiently explored.

- ..Let us think, in a given space-time, on a positioning system complemented with a number of additional clocks. This over-determined system will generate an over-determined set of data able to select a distance function with some uncertainty. Well, in spite of its interest, this problem is open for space-time distances.

- The first problem to be solved for a given distance function is if it is the geodesic distance function of a metric. I solved this problem some years ago:

Theorem 1 (structure theorem for distance functions)**:** *The necessary and sufficient condition for a distance function* $U(x,y)$ to be the geodesic distance function of a metric, is that its derivatives verify the identity:

$$U_{abc\rho}\, U^{\rho} + (U_{ab\rho}\, U_{cm\sigma} + U_{bc\rho}\, U_{am\sigma} + U_{ca\rho}\, U_{bm\sigma})\;\, U^{m\rho}\, U^{\sigma} \\ - (U_{ab\rho}\, U_{c} + U_{bc\rho}\, U_{a} + U_{ca\rho}\, U_{b})\, U_{mn\sigma}\, U^{m\rho}\, U^{n}\, U^{\sigma} = 0$$

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Theorem 2 (metric of a distance function): *In terms of the derivatives of the distance function U, the contravariant components* $g^{\alpha\beta}$ *of the metric solution at the point x are given by:*

$$g^{\alpha\beta} = U^{\alpha}\, U^{\beta} + U^{a\alpha}\, U^{b\beta}\, U_{ab\gamma}\, U^{\gamma} .$$

- These results remain, for our needs, still elemental. We must develop them, plan a phenomenology of distance functions, and be able to ask them the the finite form of the same questions that we ask to a metric.

--oo0oo--

- The finite analog of a connection remains a completely open problem.

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Message 8[1]	**Be *finite* with differential geometry.**

[1]Specially to young people

And finally, as a last message:

Message 9[1]	**Be *happy working* with relativistic positioning systems!**

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